MAT 227 8/3

Note Title 8/3/2009

2.8 #5)

$$\vec{x}' = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \vec{x}$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 2 & 1-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((1-\lambda)^2 - 1\right) - 2(1-\lambda+1) = 0$$

$$= (1-\lambda)\left[\lambda^2 - 2\lambda\right] - 2(2-\lambda) = 0$$

$$= (\lambda-2)\left[(1-\lambda)\lambda + 2\right] = 0$$

$$= -(\lambda-2)(\lambda^2 - \lambda - 2) = 0$$

$$= -(\lambda-2)^2(\lambda+1) = 0$$

$\lambda = 2$, mult 2; $\lambda = -1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = - \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$a + b + c = -a$

$2a + b - c = -b$

$-3 + 2 - 1 = -2$

$-b + c = -c$

$b = 2c$

$2a = -3$

$a = -\frac{3}{2}$

$c = 1$

$b = 2$

$\lambda = -1$ $\begin{bmatrix} -3/2 \\ 2 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$

$\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 2\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$a+b+c = 2a$$
$$2a+b-c = 2b \qquad a=0$$
$$-b+c = 2c \qquad b=1$$
$$b=-c \qquad c=-1$$

$$\lambda = 2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{x}_1 = \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix} e^{-t} \qquad \vec{x}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{2t}$$

$$\vec{x}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} t e^{2t} + \begin{bmatrix} a \\ b \\ c \end{bmatrix} e^{2t}$$

$$\vec{x}_3{}' = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} t e^{2t} + \begin{bmatrix} 2a \\ 2b \\ 2c \end{bmatrix} e^{2t}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \vec{x}_3 = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} t e^{2t} + \begin{bmatrix} a+b+c \\ 2a+b-c \\ -b+c \end{bmatrix} e^{2t}$$

$$\begin{bmatrix} 2a \\ 2b+1 \\ 2c-1 \end{bmatrix} = \begin{bmatrix} a+b+c \\ 2a+b-c \\ -b+c \end{bmatrix}$$

$b+c=a$

$1=2a-b-c \Rightarrow 2a=2$

$-1=-b-c \qquad a=1$

$b+c=1 \qquad b=1$

$b+c=0 \qquad c=0$

$$\vec{x}_3 = \begin{bmatrix} 1 \\ t+1 \\ -t \end{bmatrix} e^{2t}$$

$2 \quad -1 \qquad 1 \quad 7 \quad 1 \quad 5$

$6 \quad -5 \qquad 1 \qquad -1 \quad -5$

$\lambda = \lambda_1$ mult 2

$$\vec{x}_1 = \begin{bmatrix} \\ \\ \end{bmatrix} e^{\lambda_1 t} \qquad \vec{x}_2 = \begin{bmatrix} \\ \\ \end{bmatrix} t e^{\lambda_1 t} + \begin{bmatrix} a \\ b \\ \vdots \end{bmatrix} e^{\lambda_1 t}$$

same

7.9

3) $\vec{x}' = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \vec{x} + \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix} \qquad \vec{x}(0) = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$

$$\begin{vmatrix} 2-\lambda & -5 \\ 1 & -2-\lambda \end{vmatrix} = \lambda^2 - 4 + 5 = 0 = \lambda^2 + 1 \Rightarrow \lambda = \pm i$$

$$\begin{bmatrix} s-2 & 5 \\ -1 & s+2 \end{bmatrix} \vec{\underline{X}} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} \frac{-s}{s^2+1} \\ \frac{1}{s^2+1} \end{bmatrix}$$

$$\vec{X}(s) = \frac{1}{s^2+1}\begin{bmatrix} s+2 & -5 \\ 1 & s-2 \end{bmatrix}\left[\begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} \frac{-s}{s^2+1} \\ \frac{1}{s^2+1} \end{bmatrix}\right]$$

$$\vec{X}(s) = \begin{bmatrix} \dfrac{x_0(s+2) - 5x_1}{s^2+1} - \dfrac{s(s+2)+5}{(s^2+1)^2} \\[2ex] \dfrac{x_0 + (s-2)x_1}{s^2+1} + \dfrac{(s-2)-s}{(s^2+1)^2} \end{bmatrix}$$

$$X_1(s) = \frac{x_0(s+2) - 5x_1}{s^2+1} \boxed{- \frac{s^2+2s+5}{(s^2+1)^2}}$$

$$\downarrow$$

$$x_0 \cos(t) + 2x_0 \sin t - 5x_1 \sin t$$

$$\frac{s^2+2s+5}{(s^2+1)^2} = \frac{As}{s^2+1} + \frac{B}{s^2+1} + \frac{C(s^2-1)}{(s^2+1)^2} + \frac{D(2s)}{(s^2+1)^2}$$

$$s^2+2s+5 = \cancel{A}s(s^2+1) \overset{0}{} + B(s^2+1) + C(s^2-1) + 2Ds$$

$$B+C=1 \qquad 2D=2 \qquad B-C=5$$

$D = 1, \qquad B = 3 \qquad C = -2$

$$x_1(t) = x_0\left(\cos(t) + 2\sin t\right) - 5x_1 \sin t + 3\sin t - 2t\cos t + t\sin t$$

$$\frac{x_0 + (s-2)x_1}{s^2+1} + \frac{(s-2)-s}{(s^2+1)^2} = \frac{-2}{(s^2+1)^2} = \frac{A}{s^2+1} + \frac{B(s^2-1)}{(s^2+1)^2}$$

$$x_0\sin(t) + \left(\cos(t) - 2\sin t\right)x_1$$

$$-2 = A(s^2+1) + B(s^2-1) \Rightarrow \begin{array}{l} A + B = 0 \\ A - B = -2 \end{array} \qquad \begin{array}{l} A = -1 \\ B = 1 \end{array}$$

$$x_2(t) = x_0 \sin(t) + x_1 (\cos(t) - 2\sin t)$$

$$- \sin(t) + t\cos(t)$$

$$\vec{x}(t) = x_0 \begin{bmatrix} \cos(t) + 2\sin(t) \\ \sin(t) \end{bmatrix} + x_1 \begin{bmatrix} -5\sin t \\ \cos(t) - 2\sin(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 3\sin(t) - 2t\cos(t) + t\sin(t) \\ -\sin(t) + t\cos(t) \end{bmatrix}$$

$\lambda = \pm i$

$$\begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = i \begin{bmatrix} a \\ b \end{bmatrix}$$

$$2a - 5b = ai$$
$$a - 2b = bi$$

$D+2$ $\left(x_1' = 2x_1 - 5x_2 - \cos(t) \right)$

$$x_2' = x_1 - 2x_2 + \sin(t) \Rightarrow x_2' + 2x_2 = x_1 + \sin t$$

$$x_1'' + \underline{2x_1'} = \underline{2x_1'} + 4x_1 - 5(x_1 + \sin(t)) - \cos(t)$$

$$x_1'' + x_1 = -5\sin(t) - \cos(t)$$

$$r^2 + 1 = 0$$

$$r = \pm i$$

$$x_1 = c_1 \cos(t) + c_2 \sin(t)$$

$$x_p = At\cos t + Bt\sin(t)$$

$$x_p'' = -At\cos(t) - 2A\sin t - Bt\sin t + 2B\cos(t)$$

$$+ x_p = At\cos t \qquad + Bt\sin t$$

$$-2A\sin t + 2B\cos t = -5\sin t - \cos t$$

$A = 5/2 \qquad B = -1/2$

$x_1 =$

System $\leftrightarrow$ Higher Order

$x' = x - y \qquad x'' - 2x' + 2x = 0$

$y' = x + y$

$u = y$

$y''' - 3y'' + y = 0 \qquad v = y' = u'$

$w = v' = y''$

$$\boxed{\begin{aligned} u' &= v \\ v' &= w \\ w' &= 3w - u \end{aligned}}$$

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \vec{x} \qquad \vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-2t}$$

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \vec{x} \qquad \vec{x}(t) = c_1 \begin{bmatrix} \cos(t) \\ -\cos(t) - \sin(t) \end{bmatrix} + c_2 \begin{bmatrix} \sin(t) \\ \cos(t) - \sin(t) \end{bmatrix}$$

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \vec{x} \qquad \vec{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve using Laplace Trans.

$$\begin{bmatrix} s-1 & -1 \\ 1 & s+1 \end{bmatrix} \vec{X}(s) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{X}(s) = \frac{1}{s^2} \begin{bmatrix} s+1 & 1 \\ -1 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{s+3}{s^2} \\ \frac{2s-3}{s^2} \end{bmatrix}$$

$$\vec{x}(t) = \begin{bmatrix} 1+3t \\ 2-3t \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{vmatrix} s-1 & -1 \\ 1 & s+1 \end{vmatrix} = s^2 - 1 + 1 = s^2$$