# MAT227 7/21



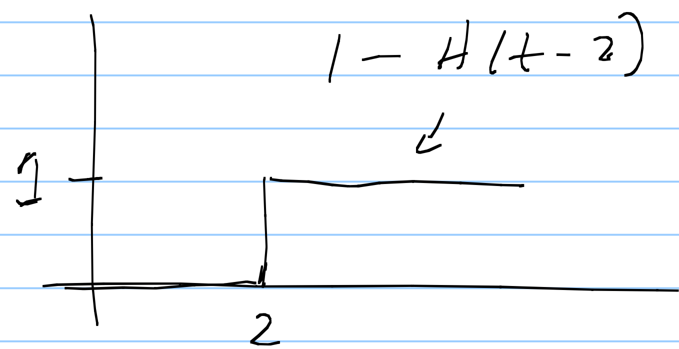
6.3)

#9) $f(t) = \begin{cases} 1 & 0 \le t < 2 \\ e^{-(t-2)} & t \geq 2 \end{cases} = 1 - H(t-2) + e^{-(t-2)} H(t-2)$

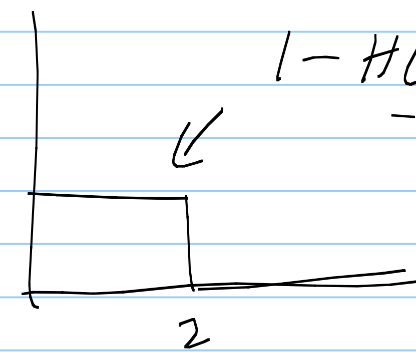
$$= 1 + H(t-2)\left(e^{-(t-2)} - 1\right)$$

$H(t-2)$

$1 - H(t-2)$




$1 - H(t-2)$



11)

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ t-1 & 1 \le t < 2 \\ t-2 & 2 \le t < 3 \\ 0 & t \geq 3 \end{cases} = t\left[1 - H(t-1)\right] + (t-1)\left[H(t-1) - H(t-2)\right] + (t-2)\left[H(t-2) - H(t-3)\right]$$

9)

$$f(t) = 1 + H(t-2)\left(e^{-(t-2)} - 1\right)$$

$$\mathcal{L}\{f(t)\} = F(s) = \frac{1}{s} + \frac{e^{-2s}}{s+1} - \frac{e^{-2s}}{s}$$

$$1 \longrightarrow 1/s$$

$$H(t-2)\, e^{-(t)-2} \longrightarrow \frac{e^{-2s}}{s+1}$$

$$-H(t-2) \longrightarrow -\frac{e^{-2s}}{s}$$

11) $f(t) = t\left[1 - H(t-1)\right] + (t-1)\left[H(t-1) - H(t-2)\right]$

$+(t-2)\left[H(t-2) - H(t-3)\right]$

$$= t + H(t-1)\left(t-1-t\right) + H(t-2)\left[t-2-(t-1)\right]$$

$$-(t-2)H(t-3)$$

$$= t - H(t-1) - H(t-2) - (t-3+1)H(t-3)$$

$$t \to 1/s^2$$

$$H(t-1) \to e^{-s}/s$$

$$H(t-2) \to e^{-2s}/s$$

$$(t-3)H(t-3) \to \frac{1}{s^2}e^{-3s}$$

$$H(t-3) \to e^{-3s}/s$$

$$F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s^2} - \frac{e^{-3s}}{s}$$

17) $$f(t) = (t-3)H(\underline{t-2}) - (\underline{t}-2)H(\underline{t-3})$$

$$= [t-2-1]\,H(t-2) - (t-3+1)\,H(t-3)$$

$$= (t-2)H(t-2) - H(t-2)$$

$$-(t-3)H(t-3) - H(t-3)$$

$$\mathcal{L}\{f\} = \frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s^2} - \frac{e^{-3s}}{s}$$

$$u_{t_0}(t) = H(t - t_0)$$

6.5 #7)

$$y'' + y = \delta(t-2\pi)\cos(t) = \delta(t-2\pi)$$

$$y(0) = 0 \quad y'(0) = 1$$

$$s^2 Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{1} + Y(s) = e^{-2\pi s}$$

$$(s^2+1)Y(s) = 1 + e^{-2\pi s}$$

$$Y(s) = \frac{1+e^{-2\pi s}}{s^2+1}$$

$$\boxed{y(t) = \sin(t) + \sin(t-2\pi)H(t-2\pi)}$$

$$\bar{Y}(s) = \frac{1+e^{-2\pi s}}{s^2+1} = \frac{1}{s^2+1} + \frac{e^{-2\pi s}}{s^2+1} \qquad \frac{1}{s^2+1}$$

$$\sin(t) + H(t-2\pi)\sin(t-2\pi)$$

15) $4y'' + 4y' + 17y = g(t)$

$y(0) = y'(0) = 0$

$$4s^2\bar{Y}(s) + 4s\bar{Y}(s) + 17\bar{Y}(s) = G(s)$$

$$\left(4s^2+4s+17\right)Y(s) = G(s)$$

$$Y(s) = \frac{G(s)}{(2s+1)^2+16} = \frac{1}{4}\,\frac{G(s)}{\left(s+\frac{1}{2}\right)^2+4} = \frac{1}{8}\,\frac{2}{\left(s+\frac{1}{2}\right)^2+4}\cdot G(s)$$

$$\mathcal{L}^{-1}\left\{\frac{2}{(s+1/2)^2+4}\right\} = \underline{e^{-1/2\,t}\sin(2t)}$$

$$y(t) = \frac{1}{8}\int_0^t e^{-1/2(t-s)}\sin(2t-2s)\,g(s)\,ds$$

$$= \frac{1}{8}\int_0^t e^{-1/2\,s}\sin(2s)\,g(t-s)\,ds$$

$$Y(s) = \frac{1}{8} \cdot \boxed{\frac{2}{(s+\frac{1}{2})^2+4}} \cdot G(s) = \underline{F(s)} \cdot \underline{G(s)}$$

$$y(t) = \int_0^t \underline{f(t-s)}\, \underline{g(s)}\, ds = \int_0^t f(s)\, \underline{g}(t-s)\, ds$$

$$f(t) \rightarrow \mathcal{L}\{f(t)\}$$

$$f(t) = te^t \qquad F(s) = \frac{1}{(s-1)^2}$$

$$\mathcal{L}\{te^t\} = \int_0^\infty te^t e^{-st}\,dt = -\frac{d}{ds}\int_0^\infty e^t e^{-st}\,dt$$

$$= -\frac{d}{ds}\mathcal{L}\{e^t\} = -\frac{d}{ds}\frac{1}{s-1} = \frac{1}{(s-1)^2}$$

$$\mathcal{L}\{\underline{\underline{t}}^2 H(\underline{t-2})\} = \left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)e^{-2s}$$

$u = \underline{t-2}$, $t = u+2$

$$t^2 H(t-2) = (u+2)^2 H(u) = (u^2 + 4u + 4)\,H(u)$$

$$\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)e^{-2s}$$

$$\mathcal{L}\{t^2 H(t-2)\} = \frac{d^2}{ds^2}\mathcal{L}\{H(t-2)\} = \frac{d^2}{ds^2}\,\frac{e^{-2s}}{s}$$

$$\mathcal{L}^{-1}\left\{\frac{2s}{s^2-7s+12}\right\} \qquad \frac{2s}{s^2-7s+12} = \frac{2s}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{D}{s-4}$$

$$A = \frac{2s}{s-4}\Big|_{s=3} = -6 \qquad B = \frac{2s}{s-3}\Big|_{s=4} = 8$$

$$= 8e^{4t} - 6e^{3t}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s^2+4}\right\} = \frac{1}{2} H(t-4)\sin(2t-8)$$

$$y'' - 7y' + 12y = 0$$

$$y(0) = 1, \quad y'(0) = 2$$

$$(s^2 - 7s + 12)\,Y(s) = s - 5$$

$$Y(s) = \frac{s-5}{(s-3)(s-4)} \qquad y(t) = 2e^{3t} - e^{4t}$$

$$y'' - 7y' + 12y = \delta(t-2)$$

$$y(0) = y'(0) = 0$$

$$y(t) = H(t-2)\left[e^{4(t-2)} - e^{3(t-2)}\right]$$

$$(s^2 - 7s + 12)\,Y(s) = e^{-2s}$$

$$Y(s) = \frac{e^{-2s}}{(s-3)(s-4)} = e^{-2s}\left[\frac{A}{s-3} + \frac{B}{s-4}\right] = \frac{e^{-2s}}{s-4} - \frac{e^{-2s}}{s-3}$$

$$A = \left.\frac{1}{s-4}\right|_{s=3} = -1, \quad B = \left.\frac{1}{s-3}\right|_{s=4} = 1$$

$$y'' - 7y' + 12y = g(t)$$

$$y(0) = y'(0) = 0$$

$$y(t) = \int_0^t g(t-s)\left[e^{4s} - e^{3s}\right] ds$$

$$u(t) + 4\int_0^t (t-s)\, u(s)\, ds = \sin(t)$$

$$\bar{U}(s) + \frac{4}{s^2}\cdot \bar{U}(s) = \frac{1}{s^2+1} = \bar{U}(s)\left[\frac{s^2+4}{s^2}\right]$$

$$\bar{U}(s) = \frac{s^2}{(s^2+1)(s^2+4)} = -\frac{1}{3(s^2+1)} + \frac{4}{3(s^2+4)}$$

$$u(t) = \frac{2}{3}\sin(2t) - \frac{1}{3}\sin(t)$$

$$\frac{s^2}{(s^2+1)(s^2+4)} = \frac{A}{s^2+1} + \frac{B}{s^2+4}$$

$$A = \left.\frac{s^2}{s^2+4}\right|_{s^2=-1} = \frac{-1}{3}$$

$$B = \left.\frac{s^2}{s^2+1}\right|_{s^2=-4} = \frac{-4}{-3} = 4/3$$