MAT 227 6/9

Note Title 6/9/2009

2.6 #7)

$$\underbrace{(e^x \sin(y) - 2y\sin(x))}_{M}dx + \underbrace{(e^x\cos y + 2\cos(x))}_{N}dy = 0$$

$$\left.\begin{array}{l} M_y = e^x\cos y - 2\sin(x) \\ N_x = e^x \cos y - 2\sin(x) \end{array}\right] =$$

$$\int M\,dx = \underline{e^x\sin(y) + 2y\cos(x) + f(y)} = \phi$$

$$\phi_y = \cancel{e^x\cos(y)} + \cancel{2\cos(x)} + f' \qquad f'(y) = 0 \to f(y) = 0$$

$$= \cancel{e^x\cos(y)} + \cancel{2\cos(x)}$$

$$e^x \sin(y) + 2y\cos(x) = c$$

#12) $$\underbrace{\frac{x\,dx}{(x^2+y^2)^{3/2}}}_{M} + \underbrace{\frac{y\,dy}{(x^2+y^2)^{3/2}}}_{N} = 0$$

$$M_y = -\frac{3}{2}x\left[x^2+y^2\right]^{-5/2}\cdot(2y) = \frac{-3xy}{\left[x^2+y^2\right]^{5/2}}$$

$$N_x = -\frac{3}{2}y\left[x^2+y^2\right]^{-5/2}(2x) = \frac{-3xy}{\left[x^2+y^2\right]^{5/2}}$$

$$\int \frac{x}{(x^2+y^2)^{3/2}}\,dx \qquad u = x^2+y^2$$

$$du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}\,du$$

$$= \frac{1}{2}\int u^{-3/2}\,du = -u^{-1/2} + f(y)$$

$$= \frac{-1}{\sqrt{x^2+y^2}} + \underline{f(y)} = \phi$$

$$\phi_y = N = \frac{y}{(x^2+y^2)^{3/2}} + f' = \frac{y}{(x^2+y^2)^{3/2}} \Rightarrow f'(y) = 0$$

$$\frac{-1}{\sqrt{x^2+y^2}} = C$$

#9) $(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x)\,dx$

$+ (xe^{xy}\cos(2x) - 3)\,dy$

$M_y = e^{xy}\cos(2x) + xye^{xy}\cos(2x) - 2xe^{xy}\sin(2x)$

$N_x = e^{xy}\cos(2x) + xye^{xy}\cos(2x) - 2xe^{xy}\sin(2x)$

$\phi = \int N\,dy = \int (xe^{xy}\cos(2x) - 3)\,dy$

$= e^{xy}\cos(2x) - 3y + f(x)$

$$\phi_x = m = ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + f'$$
$$= ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x$$

$$f'(x) = 2x \Rightarrow f(x) = x^2$$

$$\phi(x,y) = e^{xy}\cos(2x) - 3y + x^2 = C$$

Chpt 1 1–4

Chpt 2 1–6

ODE $\rightarrow$ order, linear or non-linear

$$yy'' + xy = 4xy^3$$

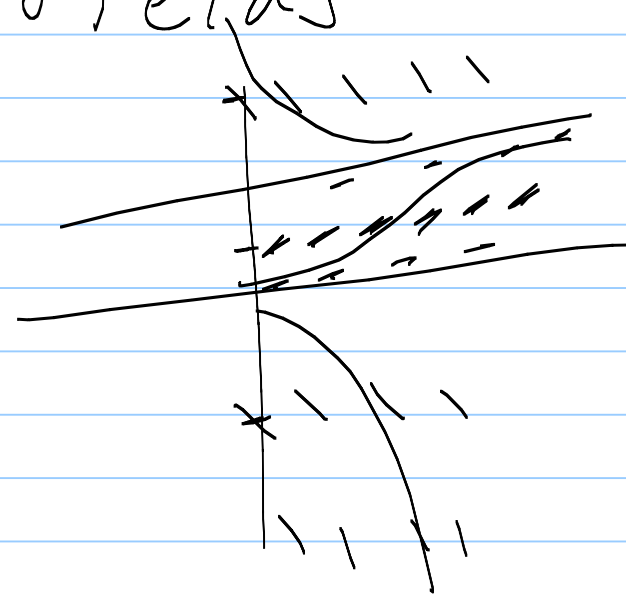
2nd order non-linear ODE

$$xy'' + xy = 4x^3y$$

2nd order linear ODE

Direction fields

$y' = y(1-y)$



$y' + P(x)y = g(x)$

$y(x_0) = y_0$

Integrating Factors

$$y'(x) + x\,y(x) = x$$

$$y(0) = 1 \qquad \mu = e^{x^2/2}$$

$$y(x) = x + e^{-x^2/2}$$

$$y' = x y^2 \Rightarrow \frac{dy}{y^2} = x\,dx$$

$$-\frac{1}{y} = \frac{x^2}{2} + c \qquad y = \frac{1}{c - x^2/2}$$

At least one application problem.

Mixture r_{in} c_{in}  r_{out}

$$\dot{Q}(t) = IN - OUT$$
$$= c_{in} r_{in} - \frac{Q}{V} r_{out}$$

$r_{in} \neq r_{out}$

$$\dot{V} = r_{in} - r_{out}$$

$y' = f(y) \rightarrow$ separable

Some exact & some integrating factor

$$2x\sin(y) + \cos(y)\,y' = 0$$

$$M \qquad\qquad N$$

$$M_y = 2x\cos(y) \qquad N_x = 0$$

$$\mu'(y) = \frac{N_x - M_y}{M}\mu = \frac{0 - 2x\cos(y)}{2x\sin(y)}\mu = -\cot(y)\,\mu$$

$$\frac{d\mu}{\mu} = -\frac{\cos(y)}{\sin(y)}dy \Rightarrow \ln(\mu) = -\ln(\sin y)$$

$$\mu = \csc(y)$$

$$2x + \cot(y)\, y' = 0$$

$$x^2 + \ln(\sin(y)) = C \rightarrow e^{x^2 + \ln(\sin(y))} = C$$

$$e^{x^2} e^{\ln(\sin(y))} = C$$

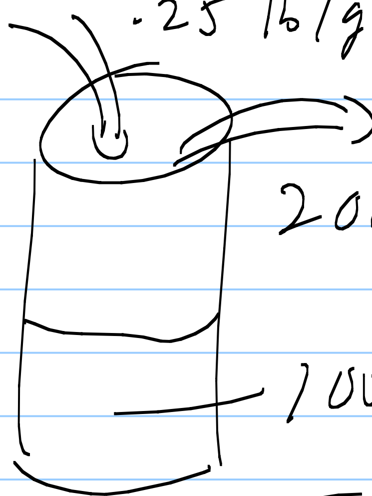
$$e^{x^2} \sin(y)$$

$$e^{x^2} \sin(y) = C$$

$$\mu'(x) = \frac{M_y - N_x}{N} \mu = \frac{2x\cos(y) - 0}{\cos(y)} \mu$$

$$= 2x\mu \Rightarrow \mu(x) = e^{x^2}$$

$$2xe^{x^2} \sin(y) + e^{x^2} \cos(y)\, y' = 0$$



2gal/min

$V(t) = 100 + 2t$ @ $t=50$

$\dot{Q} = IN - Out$

$= 4(.25) - \frac{Q}{100+2t} \cdot 2$

$Q(0) = 5$

$$\boxed{\begin{gathered}\dot{Q} = 1 - \frac{Q}{t+50} \\ Q(0) = 5\end{gathered}}$$

$$\dot{Q} + \frac{1}{t+50} Q = 1 \qquad p(t) = \frac{1}{t+50}, \int p(t) = \ln(t+50)$$

$$\mu = t+50$$

$$\left[(t+50) Q\right]^{\cdot} = t+50$$

$$(t+50) Q(t) = \frac{t^2}{2} + 50t + A$$

$$Q(0) = 5 \Rightarrow 250 = A$$

$$\boxed{Q(t) = \frac{t^2 + 100t + 500}{2t + 100}} \qquad c(t) = \frac{Q}{V}$$

$$C(t) = \frac{t^2 + 100t + 500}{(2t + 100)^2}$$

$$C(50) = \frac{(50)^2 + 100(50) + 500}{(200)^2} = 0.2$$

$$Q(t) = 40 \text{ lbs}$$

2.2 #10) $y' = \frac{1-2x}{y} \quad \underline{y}(1) = -2$

$$y\,dy = (1-2x)\,dx$$

$$\frac{y^2}{2} = x - x^2 + c$$

$$2 = 1 - 1 + c \Rightarrow c = 2$$

$$y^2 = 2x - 2x^2 + 4$$

$$y(x) = -\sqrt{4 + 2x - 2x^2}$$