Note Title 6/3/2009

MAT 227 6/3

2.3 #19) $Q(t)$ $\qquad c = \frac{Q}{V}$

$\dot{Q}(t) = \text{IN} - \text{OUT}$

$= k \cdot r + P - \frac{Q}{V} r$

$\dot{Q} + \frac{r}{V} Q = kr + P \qquad \mu = e^{r/V\, t}$

$\left[Q e^{\frac{r}{V}t}\right]^{\cdot} = (kr + p) e^{r/V\, t}$

$Q e^{\frac{r}{V}t} = \frac{V}{r}[kr + p] e^{r/V\, t} + a$

$c = \frac{Q(t)}{V} = \frac{\left[kV + \frac{pV}{r}\right] + a e^{-r/V\, t}}{V}$

$$C(t) = \underline{\underline{K + \frac{P}{r}}} + \underline{\left(C_0 - K - \frac{P}{r}\right) e^{-\frac{r}{V}t}}$$

b) $C(t) = C_0 e^{-r/V\, t} = \frac{1}{2} C_0$ $\qquad -\frac{r}{V} t = \ln\left(\frac{1}{2}\right)$

$$\underline{e^{-r/V\, t} = 1/2} \Rightarrow \underline{t = \frac{V}{r}\ln(2)} \qquad = -\ln(2)$$

$$C_0 e^{-r/V\, t} = \frac{1}{10} C_0 \Rightarrow t = \frac{V}{r}\ln(10)$$

$$\dot{Q}(t) = \underline{IN} - \underline{OUT}$$

$$F = ma$$

2.3 #11) $P(t)$ – loan balance

$$\dot{P}(t) = \text{Interest} - \text{payments} \qquad \mu = e^{-.0075t}$$

$$\dot{P} = \frac{.09}{12} P - 800\left(1 + \frac{t}{120}\right)$$

$$\dot{P} - .0075P = -800\left(1 + \frac{t}{120}\right)$$

$$\left[Pe^{-.0075t}\right]' = -800\left(1+\frac{t}{120}\right)e^{-.0075t}$$

$$Pe^{-.0075t} = \frac{-800}{-.0075}e^{-.0075t} + \frac{800t}{120 \cdot .0075}e^{-.0075t}$$

$$+ \frac{800}{120(.0075)^2}e^{-.0075t} + c$$

$$-800e^{-.0075t} \qquad \frac{-800t}{120}e^{-.0075t}$$

$$\int te^{at}\,dt = \frac{t}{a}e^{at} - \frac{1}{a^2}e^{at}$$

$$P = \frac{320000}{3} + \frac{320000}{360}t + \frac{320,000}{360(.0075)} + Ce^{.0075t}$$

$$P(0) = 100,000 = \frac{320,000}{3} + \frac{320,000}{360(.0075)} + C$$

$$C = -\frac{20,000}{3} - \frac{320,000}{360(.0075)}$$

$$P = \frac{320,000}{3} + \frac{320,000}{360}t + \frac{320,000}{360(.0075)}$$

$$-\left[\frac{20,000}{3} + \frac{320,000}{360(.0075)}\right]e^{.0075t}$$

2.2 #29) $y' = \dfrac{ay+b}{cy+d}$ $\qquad \dfrac{cy+d}{ay+b}\,dy = dx$

$$\frac{cy+d}{ay+b} = \frac{c(y+d/c)}{a(y+b/a)} = \frac{c}{a}\left[\frac{y+\frac{b}{a}-\frac{b}{a}+\frac{d}{c}}{y+b/a}\right]$$

$$= \frac{c}{a} + \frac{c}{a}\cdot\frac{\left[\frac{d}{c}-\frac{b}{a}\right]}{y-b/a} = \frac{c}{a} + \frac{da-bc}{a^2(y-b/a)}$$

$$\left(\frac{c}{a} + \frac{da-bc}{a^2(y+b/a)}\right]dy = dx$$

$$\frac{c}{a}y + \frac{da-bc}{a^2}\ln\left(y+\frac{b}{a}\right) = x + C$$

36) $(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$

$y = xv$

$dy = v\,dx + x\,dv$

$(x^2 + 3x^2 v + x^2 v^2)\,dx - x^2[v\,dx + x\,dv] = 0$

$[1 + 3v + v^2 - v]\,dx - x\,dv = 0$

$$\frac{dx}{x} = \frac{dv}{v^2+2v+1} = \frac{dv}{(v+1)^2}$$

$$\ln(x) + c = -\frac{1}{v+1} \Rightarrow \frac{1}{v+1} = C - \ln(x)$$

$$v+1 = \frac{1}{c-\ln(x)} \Rightarrow \left(v = \frac{1}{c-\ln(x)} - 1\right) x$$

$$v = \frac{y}{x} \Rightarrow y = \frac{x}{c-\ln(x)} - x$$

$$y' + p(x)\,y = q(x) \qquad \mu(x) = e^{\int_{x_0}^{x} p(t)\,dt}$$

$$y(x_0) = \underline{y_0} \qquad \mu(x_0) = 1$$

$$\int \left[y(x)\,\mu(x)\right]' = \int q(x)\,\mu(x) \qquad y(x) = \frac{1}{\mu(x)} \int_{x_0}^{x} q(t)\,\mu(t)\,dt + y_0/\mu(x)$$

$$y(x)\,\underline{\mu(x)} = \int_{x_0}^{x} q(t)\,\mu(t)\,dt + y_0$$

$$\Rightarrow y(x) = \frac{1}{\mu(x)} \int_{x_0}^{x} g(t)\,\mu(t)\,dt + y_0 / \mu(x)$$

$$\mu(x) = e^{\int_{x_0}^{x} p(t)\,dt}$$

$$y' + p(x)\,y = g(x)$$

$$y(x_0) = y_0$$

$$y' = \underline{f(x,y)}$$

$$y(\underline{x_0}) = \underline{y_0}$$

$$f, \ \frac{\partial f}{\partial y}$$

$$y' = \boxed{\frac{x}{y-1}}$$

@ $\underline{y=1}$

$$\frac{-x}{(y-1)^2}$$

$$y(\underline{0}) = \underline{0}$$

$$(y-1)\,dy = x\,dx$$

$$\left(\frac{y^2}{2} - y = \frac{x^2}{2} + c\right)2$$

$$0 = c$$

$$y^2 - 2y = x^2$$

$$y^2 - 2y - x^2 = 0$$

$$y = \frac{2 \pm \sqrt{4+4x^2}}{2} = 1 \pm \sqrt{1+x^2} \rightarrow y(x) = 1 - \sqrt{1+x^2}$$