MAT 222 6/2

Note Title 6/2/2009

2.2 #45) $y' = \dfrac{2x}{1+2y}$ $y(2) = 0$

$$\int (1+2y)\,dy = \int 2x\,dx$$

$$y + y^2 = x^2 + C \qquad x = 2,\ y = 0$$

$$0 = 4 + C \rightarrow C = -4$$

$$y + y^2 = x^2 - 4$$

1.2번

$$\left[R\dot{Q} + \frac{Q}{C} = V\right]C$$

$$RC\dot{Q} + Q = VC$$

$$RC\dot{Q} = VC - Q$$

$$\frac{RC}{VC-Q}dQ = dt$$

$$\frac{-RC\ln|VC-Q|}{-RC} = \frac{t+C}{-RC}$$

$$VC - Q = Ae^{-t/RC}$$

a) $Q(0) = 0$

$$Q(t) = VC - Ae^{-t/RC}$$

$$Q(t) = \underline{\underline{VC}}\left[1 - \underline{e^{-t/RC}}\right]$$

$\underline{\underline{\quad}}$: 정상상태 (steady state)

$\underline{\quad}$: 과도상태 (transient state) $\to 0$

2.1 #1) $\left[y' + 3y = t + e^{-2t}\right] e^{3t}$

$p(t) = 3 \Rightarrow \int p(t)\,dt = 3t$

$\mu(t) = e^{3t}$

$\left[e^{3t} y\right]' = te^{3t} + e^{t}$

$e^{3t} y = \frac{1}{3} t e^{3t} - \frac{1}{9} e^{3t} + e^{t} + C$

$y(t) = \frac{1}{3} t - \frac{1}{9} + e^{-2t} + Ce^{-3t}$

$$\int t e^{3t}\,dt \qquad u = t \qquad du = dt$$

$$dv = e^{3t}\,dt \qquad v = \frac{1}{3}e^{3t}$$

$$= \frac{1}{3}te^{3t} - \int \frac{1}{3}e^{3t}\,dt = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t}$$

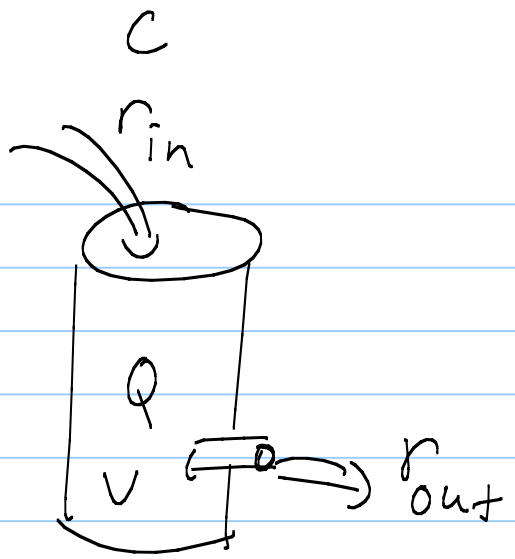
2.2 #5) $y' = \cos^2(x)\cos^2(2y)$

$$\sec^2(2y)\,dy = \cos^2(x)\,dx$$

$$\int \cos^2(x)\,dx = \int \frac{1+\cos(2x)}{2}\,dx$$

$$\frac{1}{2}\tan(2y) = \frac{1}{2}x + \frac{1}{4}\sin(2x) + c$$

$$y = \frac{1}{2}\tan^{-1}\left[x + \frac{1}{2}\sin(2x) + c\right].$$



$$\frac{dQ}{dt} = c\,r_{in} - \frac{Q}{V} r_{out}$$

$$\frac{dV}{dt} = r_{in} - r_{out}$$

$r_{in} = r_{out} \Rightarrow V$ is constant

$V = 100$ gal, $r_{in} = r_{out} = 5$ gal/min
$c = 0.05$ kg/gal

$Q(0) = 10$ kg

$$\dot{Q} = .25 - \frac{Q}{100} \cdot 5 = .25 - .05Q \quad Q(0) = 10$$

$$\dot{Q} + .05Q = .25 \qquad \mu = e^{.05t}$$

$$Q(0) = 10$$

$$\left[Qe^{.05t}\right]' = .25e^{.05t}$$

$$\Rightarrow \left(e^{.05t}Q = 5e^{.05t} + C\right)e^{-.05t}$$

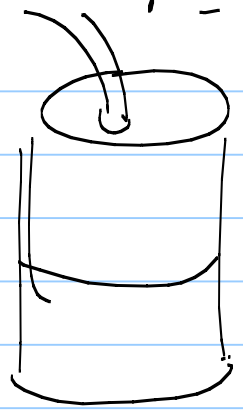
$$\Rightarrow Q = 5 + Ce^{-.05t} \qquad t = 0,\ Q = 10$$

$$10 = 5 + C \Rightarrow C = 5$$

$$Q(t) = 5 + 5e^{-.05t}$$

$c = 10 g/gal$

$r = 5 gal/min$



100 gal
half full

$Q(0) = 0$

$\frac{dV}{dt} = r_{in} = 5$

$V(t) = 50 + 5t$

$$\frac{dQ}{dt} = (10)(5) - \frac{Q}{50+5t} \cdot 0 = 50$$

$$\boxed{Q(t) = 50t} \qquad t < 10 min$$

At $t = 10$, $Q(10) = 500 g$

$$\boxed{\dot{Q} = 50 - \frac{Q}{100} \cdot 5 \qquad Q(10) = 500 g}$$

$$\left(\dot{Q} + .05Q = 50\right)e^{.05t}$$

$$\left(Qe^{.05t}\right)' = 50e^{.05t}$$

$$Q e^{.05t} = 1000 e^{.05t} + C$$

$$Q(t) = 1000 + C e^{-.05t}$$

$$Q(10) = 500 = 1000 + C e^{-.5} \Rightarrow C = -500 e^{.5}$$

$$Q(t) = 1000 - 500 e^{(.5-.05t)} \qquad t \geq 10 \text{ min}$$

$$Q(t) = \begin{cases} 50t & 0 \leq t < 10 \\ 1000 - 500 e^{(.5-.05t)} & t > 10 \end{cases}$$

$$\dot{v} = -g + \mu v^2 \qquad v_f = \sqrt{g/\mu} \qquad \omega = \sqrt{g\mu}$$

$$v(0) = 0$$

$$\boxed{v(t) = -v_f \tanh(\omega t)}$$

Hyperbolic Functions

$$\cosh(x) \equiv \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) \equiv \frac{e^x - e^{-x}}{2}$$

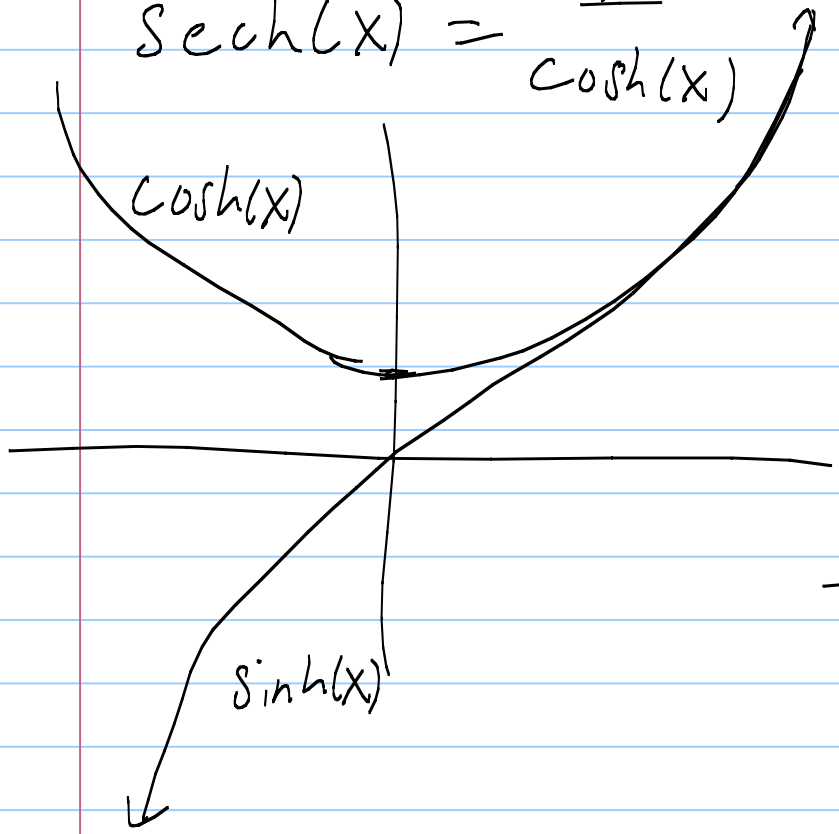
$$\cosh(x) + \sinh(x) = e^x$$

$$\cosh(x) - \sinh(x) = e^{-x}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$



$$v(t) = -v_f \tanh(\omega t)$$

$\tanh(\omega t) \sim \omega t$ (t small)

$t \to \infty \quad \tanh(\omega t) \to 1$

$t \to \infty \quad v(t) = -v_f$ Terminal Velocity

$$= -\sqrt{g/\mu}$$

$$v(t) \sim -\sqrt{\frac{g}{\mu}} \cdot \sqrt{g\mu}\, t = -gt$$

$$\int -g$$

$v(t) = -gt$

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$