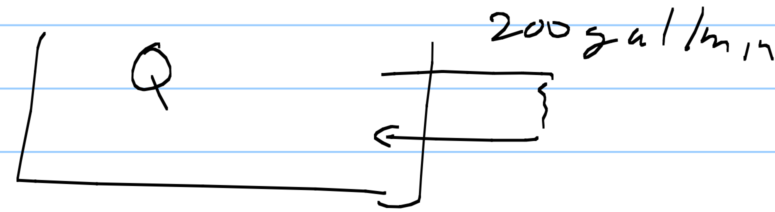
Note Title 6/1/2009

MAT 227



$$-\frac{Q}{V}\cdot r = \dot{Q}$$

$$Q(0) = 5$$

$$\dot{Q} = -\frac{200}{60{,}000} Q$$

$$Q(0) = 5$$

$$\frac{dQ}{Q} = -\frac{1}{300}\,dt$$

$$\ln(Q) = -\frac{t}{300} + C$$

$$Q = Ae^{-t/300}$$

$$\dot{Q} = -\frac{1}{300} Q$$

$$Q(0) = 5$$

$$\Rightarrow Q(t) = 5e^{-t/300}$$

$$a_n(x)\,y^{(n)} + a_{n-1}(x)\,y^{(n-1)} + \cdots + a_1(x)\,y' + a_0(x)\,y = b(x)$$

$y \to y_1 + y_2 \qquad (y_1) + (y_2)$

$$\boxed{xy'' + 4xy' + 3y = 7}$$

$y_1 y_1'' + 4xy_1' + 3y_1$

$+ y_2 y_2'' + 4xy_2' + 3y_2$

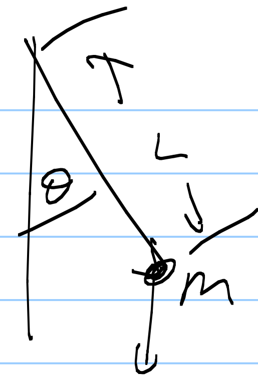
$yy'' + 4xy' + 3y = 7$

$y_1 y_2'' + y_2 y_1''$

$$x(y_1+y_2)'' + 4x(y_1+y_2)' + 3(y_1+y_2)$$

$$(xy_1'' + 4xy_1' + 3y_1) + (xy_2'' + 4xy_2' + 3y_2)$$

1.3 #31) $\frac{d}{dt} L = T$



$L = mL^2\omega$

$= mL^2 \frac{d\theta}{dt}$

$\vec{T} = \vec{r} \times \vec{F} \Rightarrow T = rF\sin\theta$

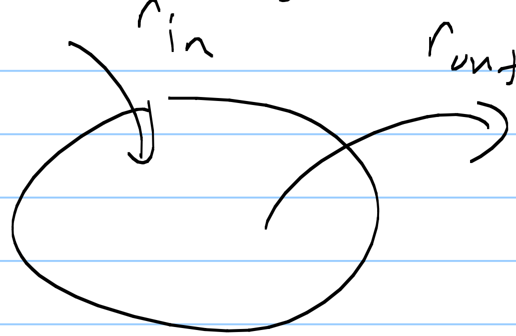
$= Lmg\sin\theta$

$\cancel{m}L^2 \frac{d^2\theta}{dt^2} = -L\cancel{m}g\sin\theta$

$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$

1.1 #21)

$c_{in} = \frac{0.01\,g}{gal}$ $\quad c_{out} = \frac{Q}{V}$



$r_{in} = r_{out} = r = 300\,gal/h$

$\dot{Q} = c_{in} r_{in} - c_{out} r_{out}$ $\qquad \dot{Q} = 3 - \frac{300}{1{,}000{,}000} Q$

$Q(0) = ??$

$\dot{Q} = \overset{10{,}000}{\cancel{\beta}} - 3 \times 10^{-4} Q$ $\qquad \frac{dQ}{Q - 10{,}000} = -.0003\,dt$

$Q(t) = \overset{10{,}000}{\cancel{\beta}} + (Q_0 - \underset{10{,}000}{\cancel{\beta}}) e^{-.0003t}$ $\qquad \ln(Q - 10{,}000) = -.0003t + c$

$Q - 10{,}000 = Ae^{-.0003t}$

$Q = 10{,}000 + Ae^{-.0003t}$

1.1 #23) $\frac{dT}{dt} \propto T_A - T \Rightarrow \frac{dT}{dt} = r(T_A - T)$

$\dot{T} = .05(70 - T) = 3.5 - .05T$

$T = 70 + (T_0 - 70)e^{-.05t}$

Separable Equations

$$y' = \underline{f(x,y)}$$

$y' + p(x)\,y = q(x) \leftarrow$ Integrating Factors

$$y' = \underline{\underline{f(x)}}\;\underline{\underline{g(y)}} \Rightarrow \underline{\underline{\frac{dy}{g(y)}}} = \underline{\underline{f(x)\,dx}}$$

$$M(x) + N(y)\,y' = 0$$

$$M(x)\,dx + N(y)\,dy = 0$$

$$\boxed{y' = -\frac{x}{y}} \qquad yy' = -x$$

$$y\,dy = -x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

$$\boxed{x^2 + y^2 = C}$$

$$y' = x(1-y^2)$$

$$\frac{dy}{y^2-1} = -x\,dx$$

$$\int \frac{dy}{y^2-1}$$

$$\frac{1}{y-1} = A\cancel{\frac{y+1}{y-1}}^{\,0} + B \Big|_{y=-1}$$

$$B = -\frac{1}{2}$$

$$\frac{1}{y^2-1} = \frac{1}{2}\left[\frac{1}{y-1} - \frac{1}{y+1}\right]$$

$$\left(\frac{1}{(y-1)(y+1)} = \frac{A}{y-1} + \frac{B}{y+1}\right)(y-1)\Big|_{y=1}$$

$$\frac{1}{2} = \frac{1}{y+1} = A + B\cancel{\frac{y-1}{y+1}}^{\,0}\Big|_{y=1}$$

$$\int \frac{dy}{y^2-1} = \frac{1}{2}\int \frac{dy}{y-1} - \frac{dy}{y+1} = \frac{1}{2}\ln(y-1) - \frac{1}{2}\ln(y+1)$$

$$= \frac{1}{2}\ln\left(\frac{y-1}{y+1}\right)$$

$$-\int x\,dx = -\frac{x^2}{2} + C$$

$$\boxed{y = \frac{1+Ae^{-x^2}}{1-Ae^{-x^2}}}$$

$$\frac{1}{2}\ln\left(\frac{y-1}{y+1}\right) = -\frac{x^3}{2} + C$$

$$\left[\frac{y-1}{y+1} = Ae^{-x^2}\right](y+1) \qquad y-1 = Ae^{-x^2}(y+1)$$

$$y = \frac{1+Ae^{-x^2}}{1-Ae^{-x^2}}$$

#(3) $y' = \dfrac{2x}{y+x^2y} = \dfrac{2x}{y(1+x^2)}$

$$\int y\,dy = \int \frac{2x}{1+x^2}\,dx$$

$$\frac{y^2}{2} = \ln(1+x^2) + C$$

$$y' = y - y^3$$

$$\frac{dy}{y-y^3} = dx$$

$$\frac{1}{y-y^3} = \frac{1}{y(1-y)(1+y)} = \frac{A}{y} + \frac{B}{1-y} + \frac{C}{1+y}$$

$$A = \frac{1}{(1-y)(1+y)}\Big|_{y=0} = 1$$

$$B = \frac{1}{y(1+y)}\Big|_{y=1} = \frac{1}{2}$$

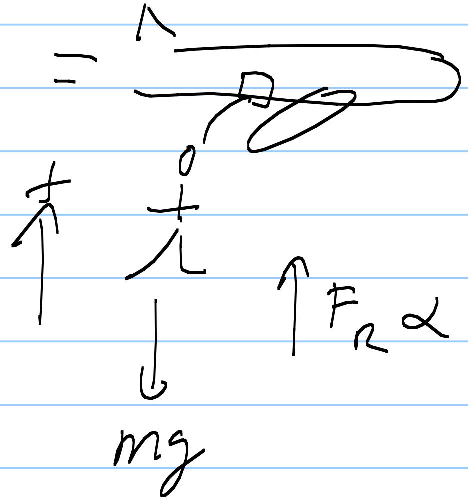
$$C = \frac{1}{y(1-y)}\Big|_{y=-1} = -\frac{1}{2}$$

$$\int \frac{dy}{y-y^3} = \int \frac{dy}{y} + \frac{1}{2}\int \frac{dy}{1-y} - \frac{1}{2}\int \frac{dy}{1+y}$$

$$= \ln(y) - \frac{1}{2}\ln(1-y) - \frac{1}{2}\ln(1+y)$$

$$= \ln\left[\frac{y}{\sqrt{1-y^2}}\right] = x + C$$

$$\frac{y}{\sqrt{1-y^2}} = Ae^x$$



$\uparrow F_R \propto v^2$

$$\underline{m}\,a = -\underline{m}g + \underline{\mu m}\,v^2$$

$$\dot{v} = -g + \mu v^2$$

$$g = \ell/t^2 \qquad \mu = \frac{1}{\ell}$$

$$\frac{g}{\mu} = \frac{\ell^2}{t^2} = v^2$$

$$\omega v_f u' = -g + \mu v_f^2 u^2$$

$$v_f = \sqrt{\frac{g}{\mu}} \qquad \omega = \sqrt{g\mu}$$

$$g u' = -g + g u^2$$

$$v = v_f u \qquad s = \omega t \qquad \frac{d}{dt} = \omega \frac{d}{ds}$$

$$u' = -1 + u^2$$

$$\int \frac{du}{u^2-1} = \int ds$$

$$u-1 = Ae^{2s}(u+1)$$

$$= Ae^{2s}\cdot u + Ae^{2s}$$

$$\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| = s + c$$

$$u(1-Ae^{2s}) = Ae^{2s}+1$$

$$\frac{u-1}{u+1} = Ae^{2s}$$

$$u(0) = 0$$

$$A = -1$$

$$u = \frac{1+Ae^{2s}}{1-Ae^{2s}} = \frac{1-e^{2s}}{1+e^{2s}} \frac{e^{-s}}{e^{-s}} = -\frac{e^{s}-e^{-s}}{e^{s}+e^{-s}}$$

$$= -\tanh(s)$$

$$V(t) = -V_f \tanh(\omega t)$$