MAT 227 5/28

Note Title 5/28/2009

Differential Equation is any equation containing a derivative.

$y' = -x\,y$

$y(0) = 1$

$y(x) = e^{-x^2/2}$

$y' = -x\,e^{-x^2/2}$

$= -x\,y$

$\frac{dy}{dx} = -xy$

$\frac{dy}{y} = -x\,dx$

$\ln|y| = -\frac{1}{2}x^2 + c$

$y = Ae^{-\frac{1}{2}x^2}$ $\quad A = e^c$

$1 = A$

$y' = 1 + y^2 \qquad \frac{dy}{dx} = 1 + y^2$

$$\frac{dy}{1+y^2} = dx$$

$\tan^{-1}(y) = x + c$

$y = \tan(x + c)$

$$y' + xy = 4x \rightarrow y' = 4x - xy$$

$$= x(4-y)$$

$$\underline{y(0)} = \underline{0}$$

$$\frac{dy}{4-y} = x\,dx$$

$$-\ln|4-y| = \frac{1}{2}x^2 + c$$

$$4 - y = Ae^{-\frac{1}{2}x^2}$$

$$y(x) = 4 - Ae^{-\frac{1}{2}x^2}$$

$$0 = 4 - A \Rightarrow A = 4$$

$$y(x) = 4\left(1 - e^{-x^2/2}\right)$$

$$y' = 4xe^{-x^2/2}$$

$$y' + xy = \cancel{4xe^{-x^2/2}} + \underline{4x} - \cancel{4xe^{-x^2/2}}$$

First Order Equations

$$y' = f(x, y)$$

Linear Equations

$$\underline{y' + p(x)\,y} = q(x)$$

Integrating Factors

$$y' = f(x) \to y(x) = \int f(x)\,dx + C$$

$$y' + p(x)\, y = q(x)$$

$$\underline{\mu(x)\, y'} + \underline{\mu(x)\, p(x)}\, y = \mu(x)\, q(x)$$

$$[\mu(x)\, y]' = \underline{\mu}\, y' + \underline{\underline{\mu'}}\, y$$

$$\mu' = \mu p$$

$$\frac{d\mu}{\mu} = p(x)\,dx \Rightarrow \mu(x) = e^{\int p(x)\,dx}$$

$y' + xy = 4x \qquad y' = f(x)$

$\mu y' + x\mu y = 4x\mu \qquad x\mu y = \mu' y$

$[\mu y]' = \mu y' + \mu' y \qquad x\mu = \mu'$

$\mu' = x\mu \Rightarrow \frac{d\mu}{\mu} = x\,dx$

$\mu(x) = e^{x^2/2}$

$e^{x^2/2} y' + x e^{x^2/2} y = 4x e^{x^2/2}$

$\left[e^{x^2/2} y\right]' = 4x e^{x^2/2}$

$$\left[e^{x^2/2} y\right]' = 4xe^{x^2/2}$$

$$\left[e^{x^2/2} y = 4e^{x^2/2} + c\right] e^{-x^2/2}$$

$$y = 4 + ce^{-x^2/2}$$

$$y' - \tan(x)\, y = \sec(x)$$

$$\mu y' \underline{- \tan(x) \mu} y = \sec(x) \mu$$

$$(\mu y)' = \mu y' \underline{+ \mu'} y$$

$$y' - \tan(x)\, y = \sec(x)$$

$$\mu y' - \tan(x)\, \mu y = \sec(x)\, \mu \leftarrow$$

$$[\mu y]' = \mu y' + \mu' y$$

$$\mu' = -\tan(x)\, \mu \qquad \frac{d\mu}{\mu} = -\tan(x)\, dx$$

$$\ln|\mu| = \ln|\cos(x)| \Rightarrow \mu(x) = \cos(x)$$

$$[\cos(x)\, y]' = 1$$

$$\cos(x)\, y = x + c \qquad y = x\sec(x) + c \cdot \sec(x)$$

$$y' = \sec(x) + x\sec(x)\tan(x) + c\sec(x)\tan(x)$$

$$\sec(x) + \cancel{x\sec(x)\tan(x)}$$
$$+ \cancel{c\sec(x)\tan(x)}$$
$$- \cancel{x\sec(x)\tan(x)} - \cancel{c\sec(x)\tan(x)}$$
$$= \sec(x)$$

$$y' + p(x)\,y = q(x)$$

$$\mu(x) = e^{\int p(x)\,dx}$$

$$\mu' = p(x)\,\mu$$

$$y' - \frac{1}{x}\,y = \frac{1}{x^2}$$

$$\mu(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln(x)} = \frac{1}{x}$$

$$\frac{1}{x}y' - \frac{1}{x^2}y = \frac{1}{x^3}$$

$$\left(\frac{1}{x}y\right)' = \frac{1}{x^3}$$

$$\frac{1}{x}y = -\frac{1}{2}\frac{1}{x^2} + C$$

$$y(x) = Cx - \frac{1}{2x}$$

$$y' - \frac{1}{x} y = \frac{1}{x^2}$$

$$\int -\frac{1}{x}\,dx = -\ln(x)$$

$$\mu(x) = e^{-\ln(x)} = \left[e^{\ln(x)}\right]^{-1} = [x]^{-1} = \frac{1}{x}$$

$$\left[\frac{1}{x} y\right]' = \frac{1}{x^3} \Rightarrow y = cx - \frac{1}{2x}$$

$$[y' + 4y = e^{3x}]\,e^{4x} \qquad [e^{4x}y]' = e^{7x}$$

$$\int 4\,dx = 4x \qquad [e^{4x}y = \tfrac{1}{7}e^{7x} + c]\,e^{-4x}$$

$$\mu = e^{4x} \qquad y(x) = \tfrac{1}{7}e^{3x} + ce^{-4x}$$

$$\boxed{e^{4x}y' + 4e^{4x}y} = e^{7x}$$

$$[e^{4x}y]' = 4e^{4x}y + e^{4x}y'$$

$(x^2y)' = x^2y' + 2xy$

$(xy)' = xy' + y$

$$\frac{xy' + 2y = x + 1}{x}$$

$$\left[y' + \frac{2}{x}y = 1 + \frac{1}{x}\right]x^2$$

$$\int \frac{2}{x}\,dx = 2\ln(x)$$

$$\mu(x) = e^{2\ln(x)} = x^2$$

$\Rightarrow x^2y' + 2xy = x^2 + x$

$[x^2y]' = x^2 + x$

$x^2y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + C$

$y(x) = \frac{x}{3} + \frac{1}{2} + \frac{C}{x^2}$