Note Title — 11/19/2009

MAT 211 11/19/09

1) $f(x) = \dfrac{x^{2/3}}{x^2+8}$ $\quad f'(x) = \dfrac{\frac{2}{3}x^{-1/3}(x^2+8) - x^{2/3}(2x)}{(x^2+8)^2}$

$\frac{2}{3}x^{-1/3}(x^2+8) - x^{2/3}(2x)$

$\frac{2}{3}x^{-1/3}\left[x^2+8-3x^2\right] = \frac{2}{3}x^{-1/3}(8-2x^2)$

$8-2x^2=0$

$2x^2=8$

$x^2=4$

$Cp \quad x=0, \pm 2$

inc dec inc decr

$-2 \quad 0 \quad 2$

rel max, rel min, rel max

incr $(-\infty,-2) \cup (0,2)$

decr $(-2,0) \cup (2,\infty)$

2) $f(x) = x^3 - 3x^2 + 1 \quad [1, 3]$

$f'(x) = 3x^2 - 6x = 3x(x-2) = 0$

$\cancel{x=0}, \quad x=2$

$x = 1, 2, 3$

$f(1) = -1$

$f(2) = -3 \leftarrow$ min

$f(3) = 1 \leftarrow$ max

3) $f(x) = x^4 - 6x^2 + 8x - 3$

$f'(x) = 4x^3 - 12x + 8 = 0$

$x^3 - 3x + 2 = 0$

$(x-1)(x^2+x-2)$

$(x-1)(x+2)(x-1) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

cp: $x = 1, -2$

b) $f''(x) = 12x^2 - 12$

$f''(1) = 0$??

$f''(-2) = 36 > 0$ $\quad -2$ rel min

c) $12x^2 - 12 = 0$

$x^2 = 1 \Rightarrow x = \pm 1$

4) a) $\lim_{x \to \infty} \dfrac{4x^3 - 3x + 5}{3x^2 + 10x + 7} = \lim_{x \to \infty} \dfrac{4x^2}{3x^2} = \dfrac{4}{3}$

$-8 \to 0$

$\cancel{\dfrac{-8}{x+2}} \le$ b) $\lim_{x \to \infty} \dfrac{8\sin(x^2)}{x+2} \le \cancel{\dfrac{8}{x+2}} \to 0 = 0$

6) $\sqrt{10}$ $\quad x=9 \quad f(x)=\sqrt{x} \qquad f'(x)=\frac{1}{2\sqrt{x}} \quad f'(9)=\frac{1}{6}$

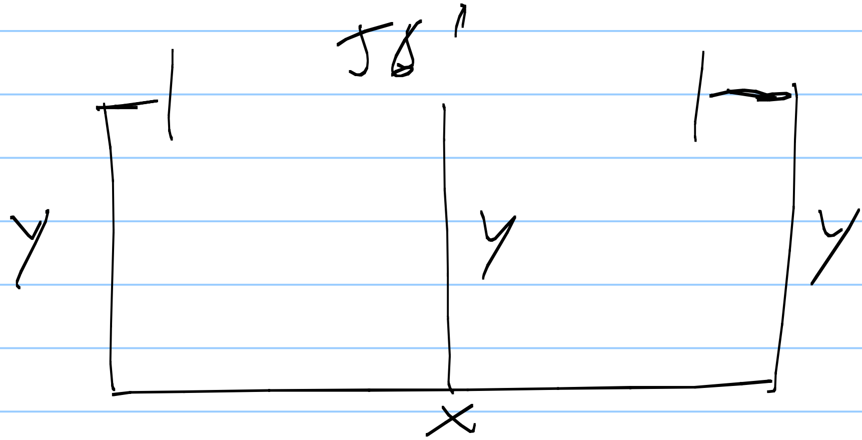
$y = f(9) + f'(9)(x-9)$

$= 3 + \frac{1}{6}(1) = 3\frac{1}{6}$

7) $y = \sin(x^2+2x)$

$dy = (2x+2)\cos(x^2+2x)\,dx$

8)



$P = 3y + x + x - 56 = 400$

$456 = 3y + 2x$

$x = \frac{456-3y}{2}$

$A = xy = y\left(\frac{456-3y}{2}\right) = 228y - \frac{3}{2}y^2$

$A' = 228 - 3y = 0 \Rightarrow y = \frac{228}{3} = 76'$

$x = \frac{456-228}{2} = \frac{228}{2} = 114'$

5) $f(x) = \frac{x^2-16a^2}{x-5a}$

a) $y_{int} = f(0) = \frac{-16a^2}{-5a} = \frac{16}{5}a$

b) $x^2 = 16a^2 \qquad x = \pm 4a$

c) $x = 5a$

d) $$f'(x) = \frac{2x(x-5a) - (x^2-16a^2)(1)}{(x-5a)^2} \quad 2,8$$

$$= \frac{x^2 - 10ax + 16a^2}{(x-5a)^2} = \frac{\overset{+}{(x-2a)}\overset{+}{(x-8a)}}{(x-5a)^2}$$

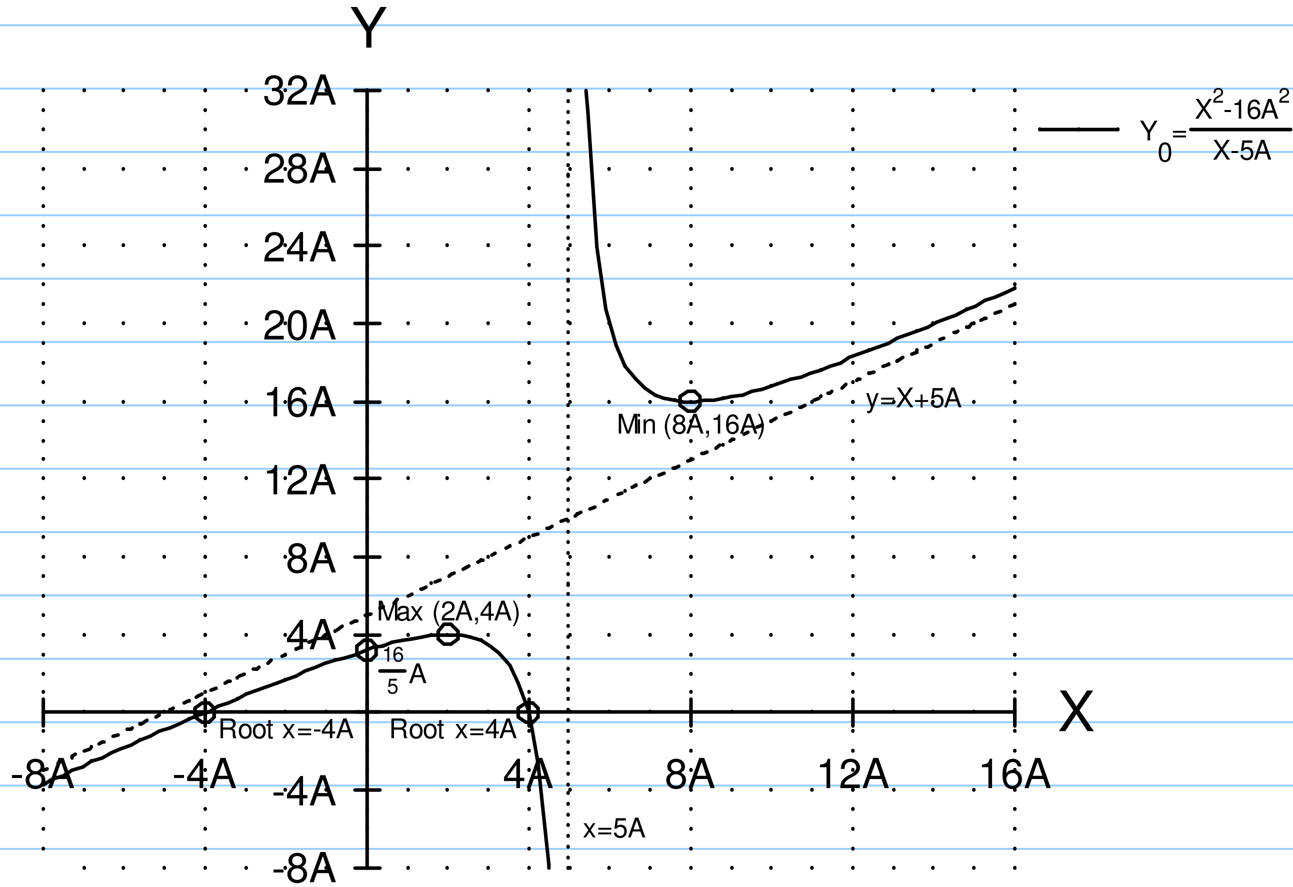
cp: $x = 5a, \; 2a, \; 8a$

rel max   rel min

+

e) $y = x + 5a$

f)



Y
X
32A
28A
24A
20A
16A
12A
8A
4A
-4A
-8A
-8A
-4A
4A
8A
12A
16A
Max (2A,4A)
Min (8A,16A)
y=X+5A
16/5 A
Root x=-4A
Root x=4A
x=5A
Y0 = (X^2-16A^2)/(X-5A)

59) $F(x) = \int_0^x t^2 \sqrt{1+t^3}\, dt$

$F'(x) = t^2 \sqrt{1+t^3} \Big|_x = x^2 \sqrt{1+x^3}$

61) $F(x) = \int_{-3}^{x} (t^2+3t+2)\, dt \quad F'(x) = x^2+3x+2$

$F(x) = \int_{-3}^{x^2} (t^2+3t+2)\, dt \Rightarrow F'(x) = (x^4+3x^2+2) \cdot 2x$

$= 2x^5 + 6x^3 + 4x$

$$\downarrow f(c_i) \qquad \downarrow dx$$

24) $\displaystyle \lim_{\|\Delta x\| \to 0} \sum_{i=1}^{n} (2c_i - 3)\,\Delta x_i \qquad \underline{[4,6]}$

$$= \int_4^6 (2x-3)\,dx$$

32) $\displaystyle \int_{-4}^{4} \sqrt{16-x^2}\,dx$

$x = 4\sin\theta \quad @x=-4,\ \theta = \frac{-\pi}{2}$

$dx = 4\cos\theta\,d\theta \quad @\,x=4,\ \theta = \frac{\pi}{2}$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} (4\cos\theta)(4\cos\theta)\,d\theta = 32\int_0^{\pi/2} \cos^2(\theta)\,d\theta$$

$$= 16\int_0^{\pi/2} [1+\cos(2\theta)]\,d\theta$$

$$= 16\left(\theta + \frac{\sin(2\theta)}{2}\right|_0^{\pi/2} = 16 \cdot \frac{\pi}{2} = 8\pi$$

$$\int_1^4 (x^2 + x - 1)\,dx$$

$$\Delta x = \frac{3}{n}$$

$$x_i = 1 + \frac{3i}{n}$$

$$\Delta A_i = \left[\left(1 + \frac{3i}{n}\right)^2 + \left(1 + \frac{3i}{n}\right) - 1\right]\frac{3}{n}$$

$$\frac{3}{n}\sum_{i=1}^{n}\left[\frac{9i^2}{n^2} + \frac{6i}{n} + 1 + \frac{3i}{n}\right]$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left[\frac{9i^2}{n^2} + \frac{9i}{n} + 1\right] = \frac{3}{n}\left[\frac{9n(n+1)(2n+1)}{6n^2} + \frac{9n(n+1)}{2n} + n\right]$$

$$= 3\left[\frac{9(n+1)(2n+1)}{6n^2} + \frac{9(n+1)}{2n} + 1\right]$$

$$\lim_{n \to \infty} 9 + \frac{27}{2} + 3 = \frac{51}{2}$$

$$\int_1^4 (x^2 + x - 1)\, dx = \frac{x^3}{3} + \frac{x^2}{2} - x \Big|_1^4 = \frac{63}{3} + \frac{15}{2} - 3$$

$$= \frac{51}{2}$$

$$\int \sec(x)\, dx = \int \frac{\sec(x)\tan(x)}{\tan(x)} = \int \frac{\sec(x)\tan(x)}{\sqrt{\sec^2(x) - 1}}\, dx$$

$u = \sec(x)$

$du = \sec(x)\tan(x)\, dx$

$$= \int \frac{du}{\sqrt{u^2 - 1}}$$

$u = \cosh(t)$

$du = \sinh(t)\, dt$

$$I = \int \frac{\sinh(t)\,dt}{\sinh(t)} = \int dt = t + C$$

$$= \cosh^{-1}(u) + C = \cosh^{-1}(\sec(x)) + C$$

$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$I = \ln\left|\sec(x) + \tan(x)\right| + C$$

75) $\int \left[1 + \sec(\pi x)\right]^2 \underline{\sec(\pi x)\tan(\pi x)\,dx}$ $u = 1 + \sec(\pi x)$

$du = \pi \sec(\pi x)\tan(\pi x)\,dx$

$$\int u^2 \frac{du}{\pi} = \frac{1}{3\pi} u^3 + C$$

$$= \frac{1}{3\pi}\left(1+\sec(\pi x)\right)^3 + C$$

15) $\xrightarrow[\text{30 sec}]{}$ 3600 ft

$$\bar{v} = \frac{v_f + v_0}{2} \Rightarrow v_f = 2\bar{v} - v_0$$
$$= 240 - 0$$
$$= 240 \text{ fps}$$

I) $\int_a^b f(x)\,dx = F(b) - F(a)$ $\quad F'(x) = f(x)$

II $F(x) = \int_a^{g(x)} f(x)\,dx$ $\quad F'(x) = f(g(x))\,g'(x)$

$\sqrt{a^2 - x}$ $\quad x = a\sin\theta$ $\quad \sqrt{a^2 + x^2}$ $\quad x = a\sinh(t)$