MAT 211 9/25

Note Title 9/25/2009

19) $f(x) = x^3 - 12x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - 12(x+h) - x^3 + 12x}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - 12x - 12h - x^3 + 12x}{h}$$

$$= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 - 12h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3hx + h^2 - 12 = 3x^2 - 12$$

81) $f(x) = \frac{1}{x+1}$ diff $x \neq -1$

91) $f(x) = |x-1|$

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^-} \frac{|x-1| - 0}{x-1} = \lim_{x \to 1^-} \frac{-(x-1)}{(x-1)} = -1$$

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^+} \frac{|x-1|}{x-1} = \lim_{x \to 1^+} \frac{x-1}{x-1} = 1$$

33) $f(x) = x^3$ $\qquad 3x - y + 1 = 0 \Rightarrow y = 3x + 1$

$m = 3$

$f'(x) = 3x^2 = 3$

$x^2 = 1 \Rightarrow x = \pm 1$

$y = 3(x-1) + 1$

$y = 3(x+1) - 1$

32) $f(x) = \frac{1}{x+1}$ $(0, 1)$

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{\frac{1}{h+1} - 1}{h} \quad \frac{(h+1)}{(h+1)}$$

$$= \lim_{h \to 0} \frac{1 - h - 1}{h(h+1)} = \lim_{h \to 0} \frac{-h}{h(h+1)} = \lim_{h \to 0} \frac{-1}{h+1} = -1$$

$$y = -1(x - 0) + 1 = -x + 1$$

$f(x) = \frac{1}{x+1}$ $f'(x) = \frac{-1}{(x+1)^2}$ $f'(0) = -1$

$$-8 \quad +8 \quad +1$$

73) $f(x) = x^3 + 2x^2 + 1 \qquad c = -2$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} = \lim_{x \to -2} \frac{x^3 + 2x^2 + 1 - 1}{x + 2}$$

$$= \lim_{x \to -2} \frac{x^2(x+2)}{(x+2)} = \lim_{x \to -2} x^2 = 4$$

21) $f(x) = \dfrac{1}{x-1}$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{1}{x+h-1} - \dfrac{1}{x-1}}{h} \quad \frac{(x-1)(x+h-1)}{(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{x-1-(x+h-1)}{h(x-1)(x+h-1)} = \lim_{h \to 0} \frac{\cancel{x}\cancel{-1}\cancel{-x}-h\cancel{+1}}{h(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{-h}{h(x-1)(x+h-1)} = \lim_{h \to 0} \frac{-1}{(x-1)(x+h-1)} = \frac{-1}{(x-1)^2}$$

36) $f(x) = \dfrac{1}{\sqrt{x-1}}$ $\qquad x + 2y + 7 = 0 \Rightarrow y = -\frac{1}{2}x - \frac{7}{2}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h-1}} - \frac{1}{\sqrt{x-1}}}{h} \cdot \frac{\sqrt{x-1}\sqrt{x+h-1}}{\sqrt{x-1}\sqrt{x+h-1}} = \lim_{h \to 0} \frac{\sqrt{x-1} - \sqrt{x+h-1}}{h\sqrt{x-1}\sqrt{x+h-1}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x-1} - \sqrt{x+h-1}}{h\sqrt{x-1}\sqrt{x+h-1}} \cdot \frac{\sqrt{x-1} + \sqrt{x+h-1}}{\sqrt{x-1} + \sqrt{x+h-1}} = \lim_{h \to 0} \frac{x-1-x-h+1}{h\sqrt{x-1}\sqrt{x+h-1}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x-1}\sqrt{x+h-1}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)} = \frac{-1}{2(x-1)^{3/2}} = -\frac{1}{2}$$

$(x-1)^{3/2} = 1$

$x - 1 = 1$

$x = 2$

$f(2) = 1 \quad (2, 1)$

$y = -\frac{1}{2}(x-2) + 1 = -\frac{1}{2}x + 2$

Rules for Derivatives

$$\frac{d}{dx}\left[c f(x)\right] = c f'(x)$$

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}e^x = e^x$$

$$f(x) = 7x^4 + 3x^2 - 9x + 5$$

$$f'(x) = 28x^3 + 6x - 9$$

$$\frac{d}{dx}7x^4 = 7 \cdot \frac{d}{dx}x^4 = 7(4x^3)$$

$$= 28x^3$$

$$\frac{d}{dx}(3x^2) = 3\frac{d}{dx}(x^2) = 3(2x)$$

$$= 6x$$

$$f(x) = 4\cos(x) - 3\sin(x) + 5e^{x}$$

$$f'(x) = -4\sin(x) - 3\cos(x) + 5e^{x}$$

$$g(x) = \frac{x-1}{\sqrt{x}} \qquad g'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$$

$$= \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$

$$= x^{1/2} - x^{-1/2}$$

$h(t) = 8 - 16t^2$ $\quad$ t in sec, h in ft

$$\frac{\Delta d}{\Delta t} \quad \lim_{\Delta t \to 0} \frac{\Delta h}{\Delta t} = v(t) \qquad \lim_{\Delta t \to 0} \frac{h(t+\Delta t) - h(t)}{\Delta t} = h'(t)$$

$v(t) = h'(t) = -32t$ $\quad$ fps

$a(t) = v'(t) = \underline{\underline{-32}}$ $\quad$ ft/sec^2

$$h(t) = h_0 + v_0 t - \frac{1}{2} g t^2$$

$$v(t) = v_0 - g t$$

$$a(t) = -g$$