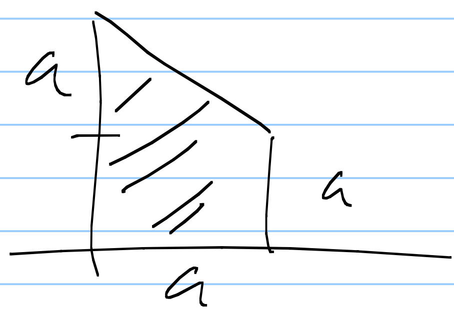
Note Title 7/30/2009

EGR180 7/30

Region I_x, I_y, I_{xy}

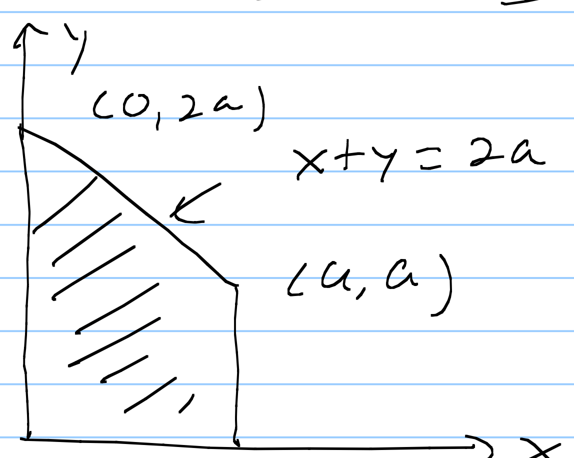


Max & Min I_x

Principal Axis

Radius of Gyration

Volume $\quad I_x,\ I_y,\ I_z,\ I_{xy},\ I_{yz},\ I_{xz}$



$$I_x = \iint_R y^2\, dA = \int_0^a \int_0^{2a-x} y^2\, dy\, dx$$

$$= \int_0^a \frac{y^3}{3}\Big|_0^{2a-x} dx = \int_0^a \frac{(2a-x)^3}{3}\, dx$$

$$= \frac{(2a-x)^4}{12}\Big|_a^0 = \frac{(2a)^4}{12} - \frac{a^4}{12} = \frac{15}{12}a^4$$

$$= \frac{5}{4}a^4$$

$$I_y = \iint_R x^2\,dA = \int_0^a \int_0^{2a-x} x^2\,dy\,dx = \int_0^a x^2(2a-x)\,dx$$

$$= \int_0^a (2ax^2 - x^3)\,dx = \frac{2ax^3}{3} - \frac{x^4}{4}\Big|_0^a = \frac{2a^4}{3} - \frac{a^4}{4}$$

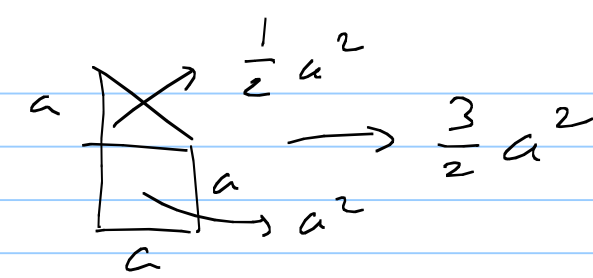
$$= \frac{5a^4}{12}$$

$$I_{xy} = \iint_R xy\,dA = \int_0^a \int_0^{2a-x} xy\,dy\,dx = \int_0^a \frac{x}{2} y^2\Big|_0^{2a-x} dx$$

$$= \int_0^a \frac{x}{2}(2a-x)^2\,dx = \frac{1}{2}\int_0^a (4a^2x - 4ax^2 + x^3)\,dx$$

$$= \frac{1}{2}\left[2a^4 - \frac{4}{3}a^4 + \frac{a^4}{4}\right. = \frac{24-16+3}{24}a^4 = \frac{11}{24}a^4$$

$$K_x = \sqrt{\frac{I_x}{A}} = \left[\frac{\frac{5}{4}a^4}{\frac{3}{2}a^2}\right]^{1/2}$$



$$= \sqrt{\frac{5}{3}}\, a$$

$$K_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}a^4}{\frac{3}{2}a^2}} = \sqrt{\frac{5}{18}}\, a$$

$$I_{max} = \frac{I_x + I_y}{2} + \left[\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2\right]^{1/2}$$

$$= \frac{1}{2}\left[\underbrace{\frac{5}{4}a^4 + \frac{5}{12}a^4}_{\frac{20}{12}}\right] + \left[\frac{1}{4}\left(\underbrace{\frac{5}{4} - \frac{5}{12}}_{\frac{10}{12}\ \frac{5}{6}}\right)^2 a^8 + \left(\frac{11}{24}\right)^2 a^8\right]^{1/2}$$

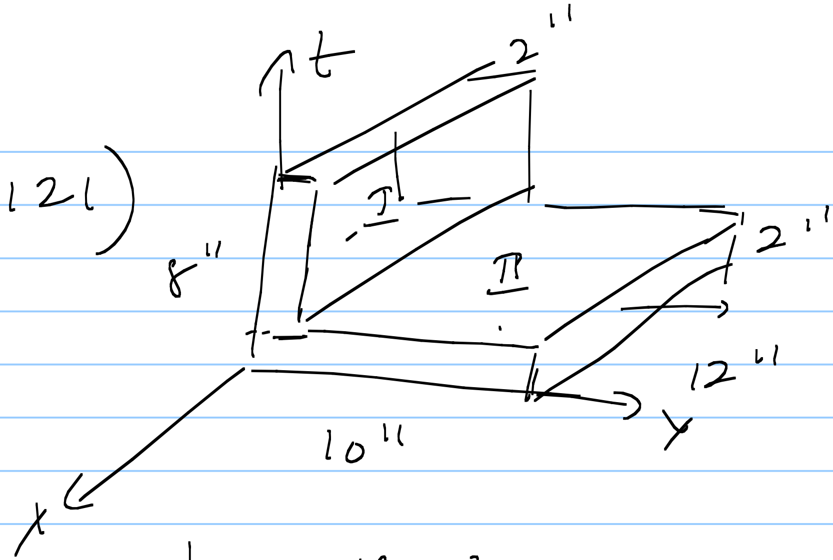
$$= \frac{5}{6} a^4 + \left[\frac{1}{4} \cdot \frac{25}{36} + \frac{121}{576} \right]^{1/2} a^4$$

$$= \frac{5}{6} a^4 + \frac{\left[\overset{144}{100} + 121 \right]^{1/2}}{24} a^4 = \left(\frac{5}{6} + \frac{\sqrt{221}}{24} \right) a^4 \doteq 1.45 a^4$$

$$I_{min} = \left(\frac{5}{6} - \frac{\sqrt{221}}{24} \right) a^4 \doteq 0{,}21\, a^4$$

$$\tan(2\theta_p) = \frac{2 I_{xy}}{I_x - I_y}$$

10-121)

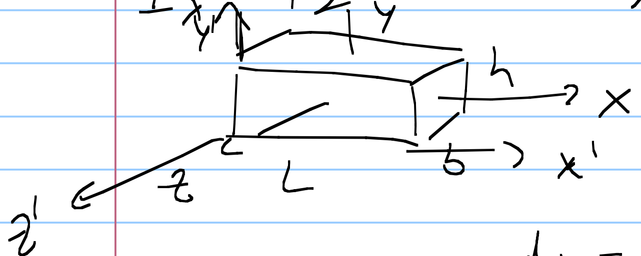


$$I_x = \iiint_V (y^2+z^2)\,\rho\, dV$$

$$= I_{x_I} + I_{x_{II}}$$

I_{x_I}

$I_x = \frac{1}{12} m\,(b^2+h^2)$ $\quad I_y = \frac{1}{12} m\,(b^2+L^2)$ $\quad I_z = \frac{1}{12} m\,(h^2+L^2)$



$I_{x'} = I_x + m\cdot[d_y^2 + d_z^2] = \frac{1}{2} m\,[b^2+h^2]$

$d_x = \frac{b}{2}$

$(\frac{b}{2})^2 + (\frac{L}{2})^2$

$$I_{x_{II_c}} = \frac{m}{12}(100+4) = \frac{104}{12}m$$

$$I_{x_{II}} = \frac{104}{12}m + m\left[\underset{6}{d_y^2} + \underset{1}{d_z^2}\right] = \frac{104}{12}m + 37m = \frac{153}{12}m$$

$$m = \rho \cdot V = 460 \cdot 10 \cdot 12 \cdot 2 = 110{,}400 \text{ lbs}$$

$$I_{x_{II}} = 814.6 \quad \text{lbs} \cdot \text{ft}^2$$

$$I_{x_{I_c}} = \frac{m}{12}\left[b^2 + h^2\right] = \frac{m}{12}\left(2^2 + 6^2\right) = \frac{40m}{12}$$

$$I_{x_I} = \frac{40m}{12} + m\left(\underset{6}{d_y^2} + \underset{5}{d_z^2}\right) = \frac{40m}{12} + m(36+25) = \frac{40}{12}m + 61m$$

$$= \underline{4116.2} \quad \text{lbs} \cdot \text{ft}^2$$