EGR 180 7/29/10

Note Title 7/29/2010

Equilibrium & Stability

$\frac{d}{ds} U(s) = 0$, values of s are equilibrium points, s_0

$\frac{d^2}{ds^2} U(s_1) > 0 \rightarrow$ stable

$< 0 \rightarrow$ unstable

$= 0 \rightarrow$ neutral

@ $\theta = 0$, $U'' = kL^2 - 2FL > 0 \Rightarrow kL > 2F$

@ $\theta = \pi$, $U'' = kL^2 + 2FL > 0 \Rightarrow$ always stable

@ $\cos\theta = \frac{2F}{kL}$

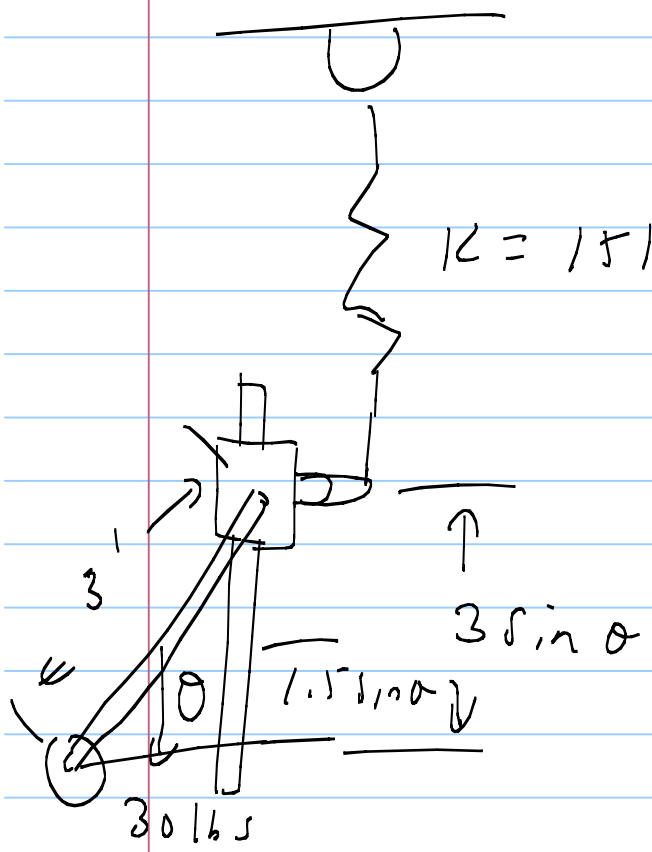
$U'' = kL^2(2\cos^2\theta - 1) - 2FL\cos\theta$

$= kL^2\left(\frac{8F^2}{k^2L^2} - 1\right) - 2FL\left(\frac{2F}{kL}\right)$

$= \frac{4F^2}{k} - kL^2 > 0$ $\qquad \frac{4F^2}{k} > kL^2$

$4F^2 > k^2L^2$

$2F > kL$



$$\Delta x = 3 - 3\sin\theta$$

$$U = \frac{1}{2}(15)(3-3\sin\theta)^2 + 30(1.5\sin\theta)$$

$$U = 67.5(1-\sin\theta)^2 + 45\sin\theta$$

$$U' = 135(1-\sin\theta)(-\cos\theta) + 45\cos\theta = 0$$

$$= 45\cos\theta\left[3\sin\theta - 2\right] = 0$$

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$

$\sin\theta = 2/3$

$U'' = -45\sin\theta\left[3\sin\theta - 2\right) + 45\cos\theta(3\cos\theta)$

$135\cos^2\theta = 135(1-\sin^2\theta)$

@ $\theta = 90^\circ$

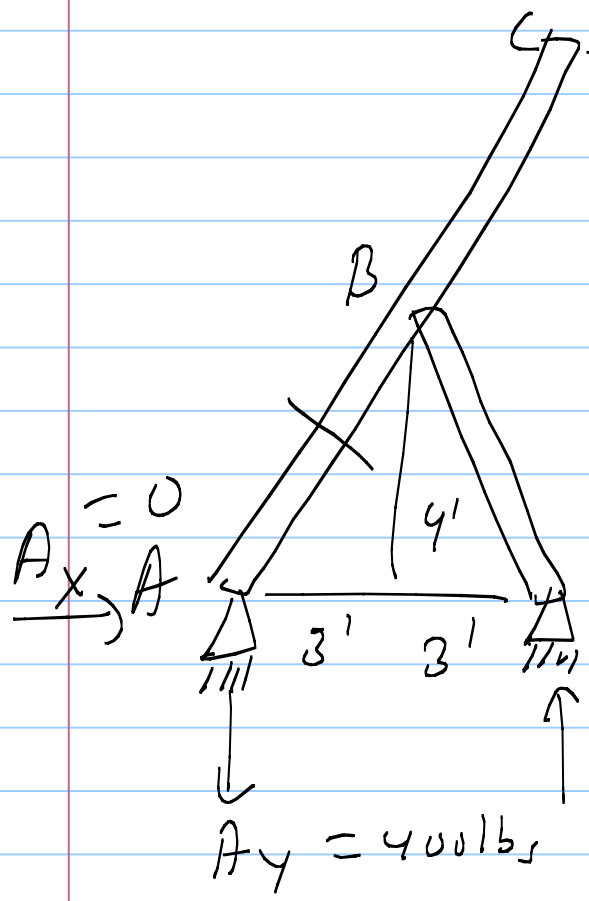
$U'' = (-45)(1) < 0$ unstable

@ $\sin\theta = 2/3$

$U'' = 135\left(1 - \frac{4}{9}\right) = 135 \cdot \frac{5}{9} > 0$ stable

$\theta = \sin^{-1}(2/3) = 41.8^\circ$

Exam 6

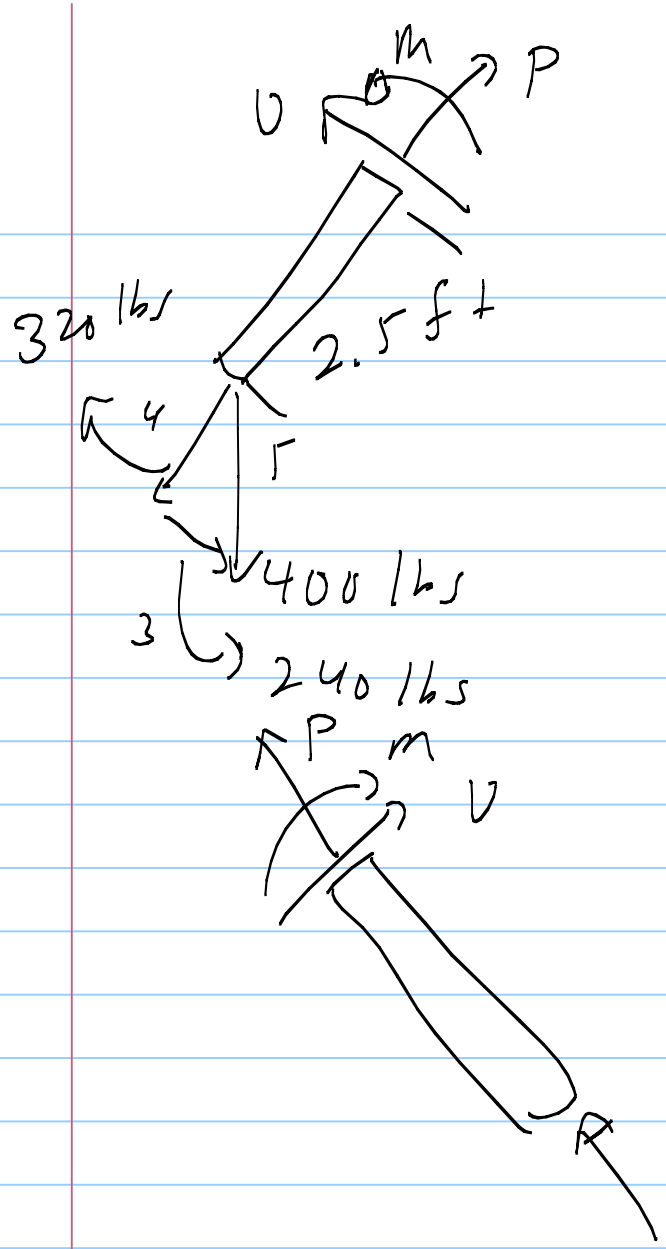


$\sum M_A = 6D_y - 8 \cdot 300 = 0$

$D_y = 400$ lbs

$D \leftarrow D_x = 300$ lbs $\quad \frac{D_x}{D_y} = \frac{3}{4}$

$D_x = 300$ lbs



$P = 320$ lbs

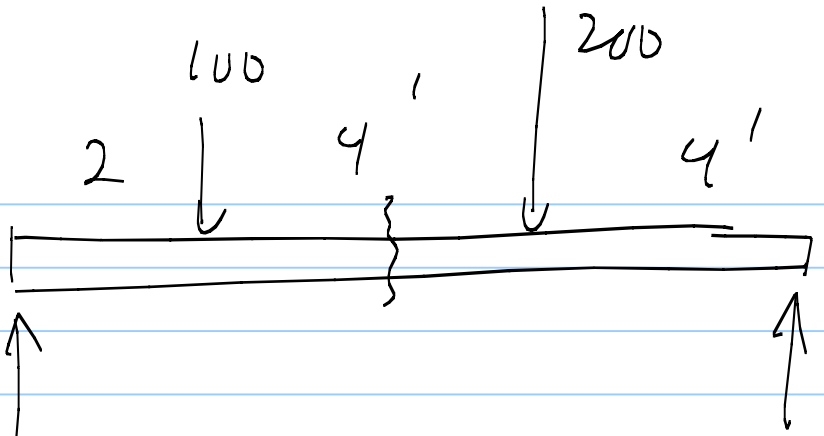
$V = 240$ lbs

$2.5(240) + m = 0 \Rightarrow m = -600$ ft-lbs

$V = 0$, $m = 0$

$P = -500$ lbs

2)



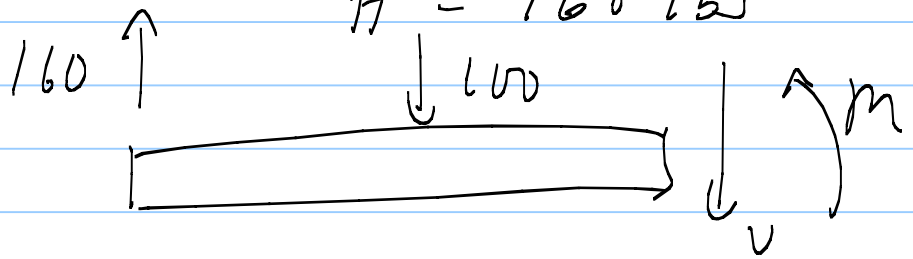
A = 160 lbs B = 140 lbs

$$\sum M_A = 10B - 200 - 1200 = 0$$

$$B = 140 \text{ lbs}$$

$$\sum F_y = A - 100 - 200 + 140 = 0$$

$$A = 160 \text{ lbs}$$

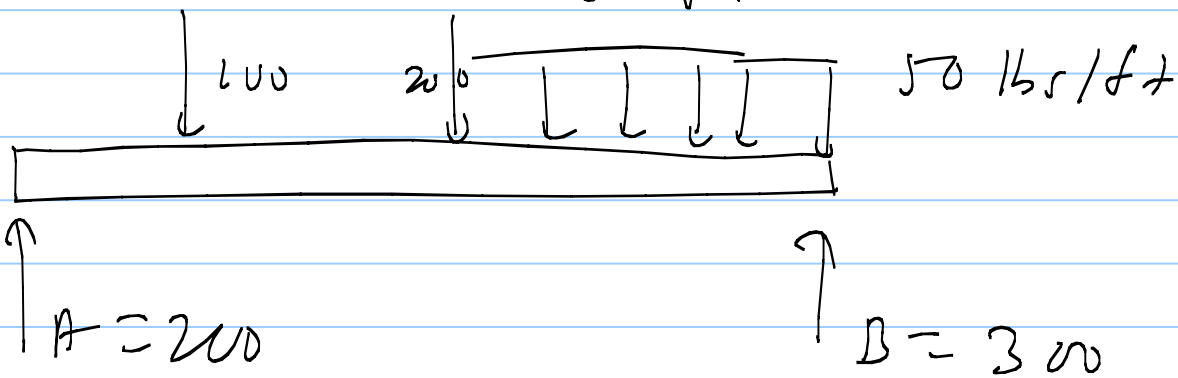


$$\sum F_y = 160 - 100 - V = 0 \Rightarrow V = -60 \text{ lbs}$$

$$\sum M_A = -2(100) - 5(-60) + M = 0$$

$$M = -100 \text{ ft·lbs}$$

3)



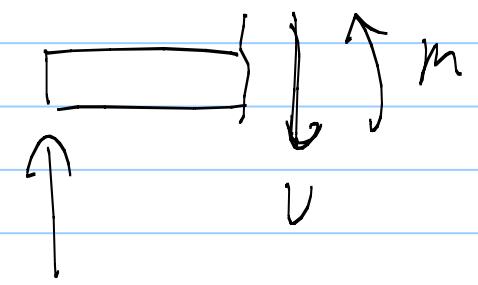
$$\sum M_A = -2(100) - 6(200) - 8(200) + 10B = 0$$

$$10B = 200 + 1200 + 1600 = 3000$$

$$B = 300 \text{ lbs}$$

$$\sum F_y = A - 100 - 200 - 200 + 300 = 0$$

$$\Rightarrow A = 200 \text{ lbs}$$



$$V(x) = \begin{cases} 200 & 0 < x < 2 \\ 100 & 2 < x < 6 \\ -100 - 50(x-6) & 6 < x < 10 \end{cases}$$

$$200 - 50x$$

$$m(x) = \begin{cases} 200x & 0 < x < 2 \\ 100x + 200 & 2 < x < 6 \\ 200x - 25x^2 + 500 & 6 < x < 10 \end{cases}$$

M(x)
V(x)
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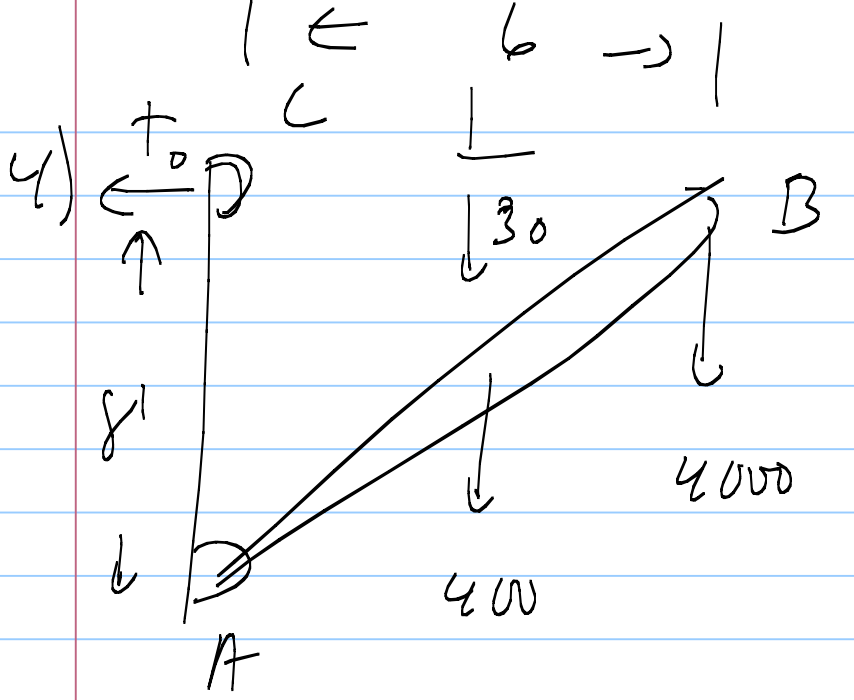


$$\sum M_A = -3(430) - 6(4000)$$

$$+8T_0 = 0$$

$$8T_0 = 25290$$

$$T_0 = 3161.25 \text{ lbs}$$

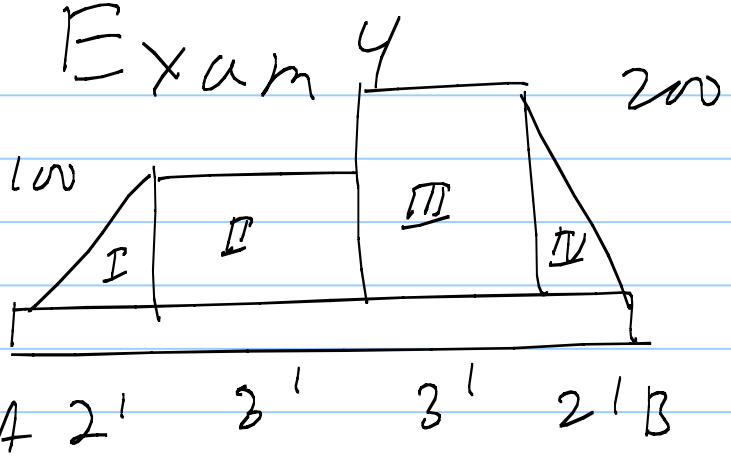
$$y(x) = \frac{T_0}{w}\left[\cosh\left(\frac{w}{T_0}x\right) - 1\right]$$

$$= \frac{3161.25}{5}\left[\cosh\left(\frac{15}{3161.25}\right) - 1\right]$$

$$= .007 \text{ ft}$$

Exam 4

1)



$F_I = 100$ lbs $x_I = \frac{4}{3}$ ft

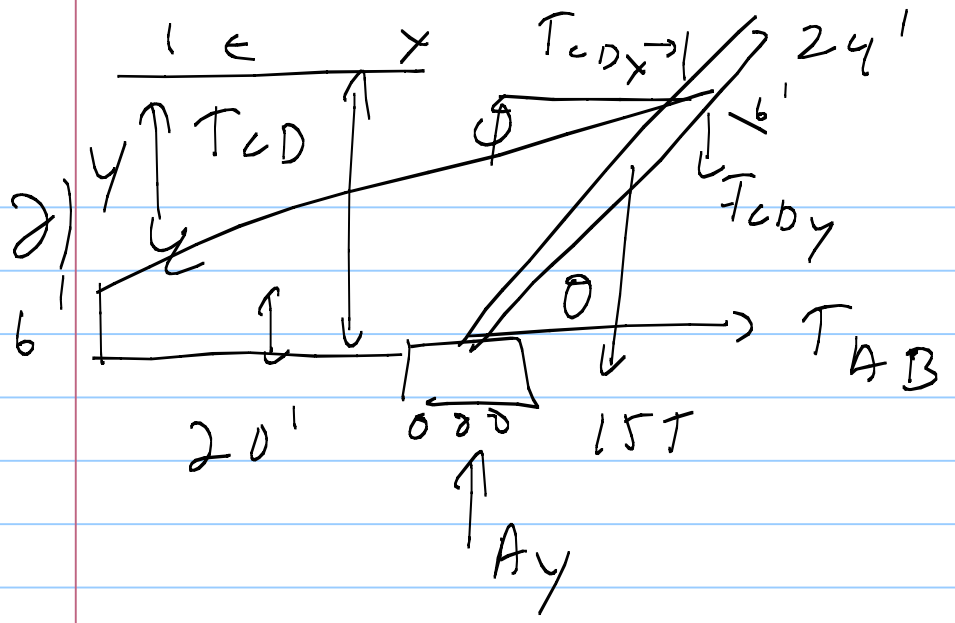
$F_{II} = 300$ lbs $x_{II} = 3.5$ ft

$F_{III} = 600$ lbs $x_{III} = 6.5$ ft

$F_{IV} = 200$ lbs $x_{IV} = \frac{26}{3}$ ft

$$\sum M_A = -100\left(\tfrac{4}{3}\right) - 300(3.5) - 600(6.5) - 200\left(\tfrac{26}{3}\right) + 10B = 0$$

$B = 681.7$ lbs $\quad A = 518.3$ lbs



$$\sum M_A = -12\cos\theta\, W + 18\sin\theta\,(T_{CD}\cos\phi) - 18\cos\theta\,(T_{CD}\sin\phi) = 0$$

$$18\sin(\theta-\phi)\,T_{CD} = 12W\cos\theta$$

$$\sum F_x = T_{AB} - T_{CD}\cos\phi = 0 \qquad T_{CD} = \frac{12W\cos\theta}{18\sin(\theta-\phi)}$$

$$T_{AB} = T_{CD}\cos\phi = \frac{12W\cos\theta\cos\phi}{18\sin(\theta-\phi)}$$

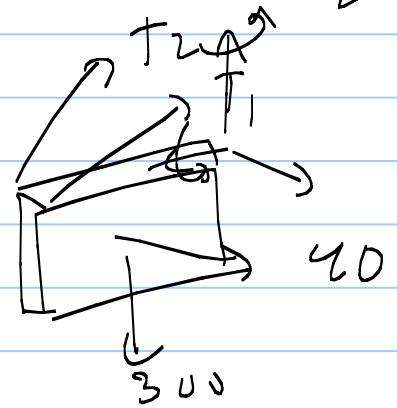
$$y = 18\sin\theta - 6 \qquad x = 20 + 18\cos\theta$$

$$\frac{y}{x} = \tan\varphi = \frac{18\sin\theta - 6}{20 + 18\cos\theta}$$

$$T_{AB} = \frac{12W\cos\theta}{18\sin\theta - 18\cos\theta\tan\varphi}$$

$$T_{AB} = \frac{12W\cos\theta}{18\left[\sin\theta - \cos\theta\,\frac{18\sin\theta - 6}{20 + 18\cos\theta}\right]}$$

3)



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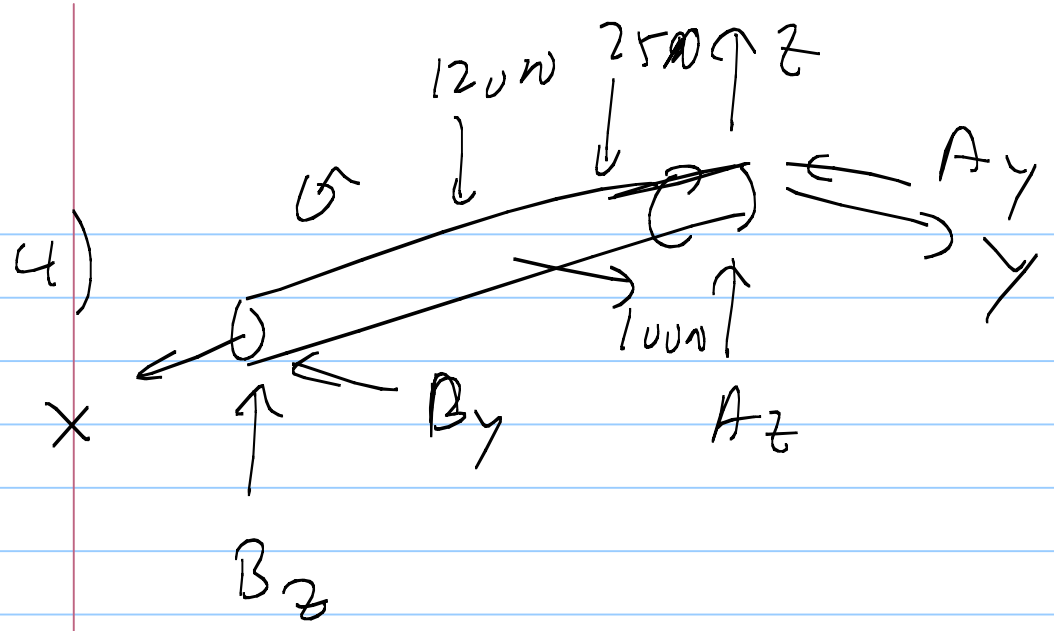

$$\sum M_{B_z} = 100 - 3 \cdot 100 + 5A_y = 0$$

$$5A_y = 200 \Rightarrow A_y = 40\ N,\ B_y = 60\ N$$

$$\sum M_{B_y} = -75 - 2(120) - 4(75) + 5A_z = 0$$

$$A_z = 123\ N,\ B_z = 72 N$$