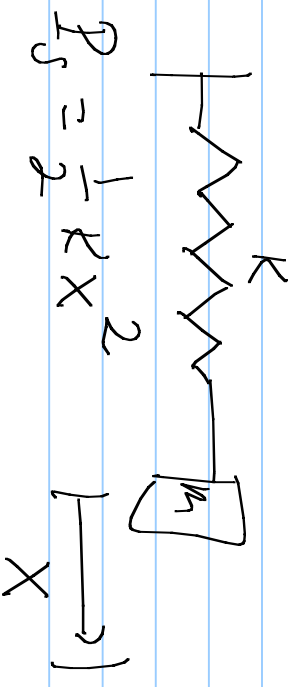
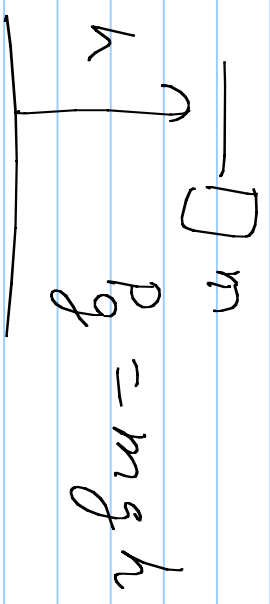


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Conservative Forces

Gravitational

Elastic

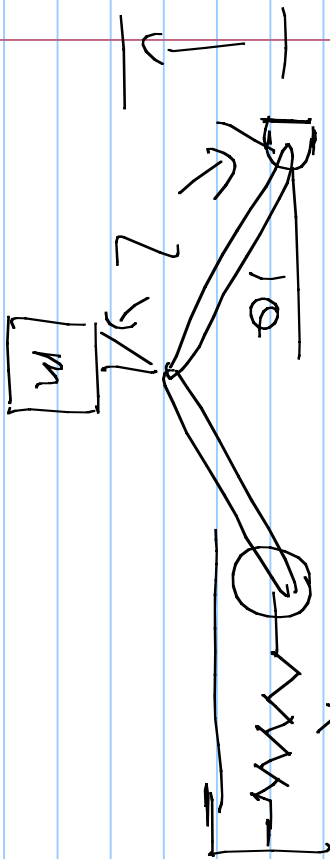


$$\delta W + \delta P = 0$$

$$\delta P = \frac{dP}{ds} \delta s = 0 \quad \frac{dP}{ds} = 0$$

$$l \in 2L \quad x \rightarrow 1$$

$$l \in 2L \cos \theta \rightarrow 1 \quad k$$



Assume rest @ $\theta = 0$

$$y = L \sin \theta$$

$$x = 2L - 2L \cos \theta$$

$$P_g = -mgL \sin \theta$$

$$P_s = \frac{1}{2} k [2L(1 - \cos \theta)]^2$$

$$P = P_g + P_s = 2kL^2(1 - \cos \theta)^2 - mgL \sin \theta$$

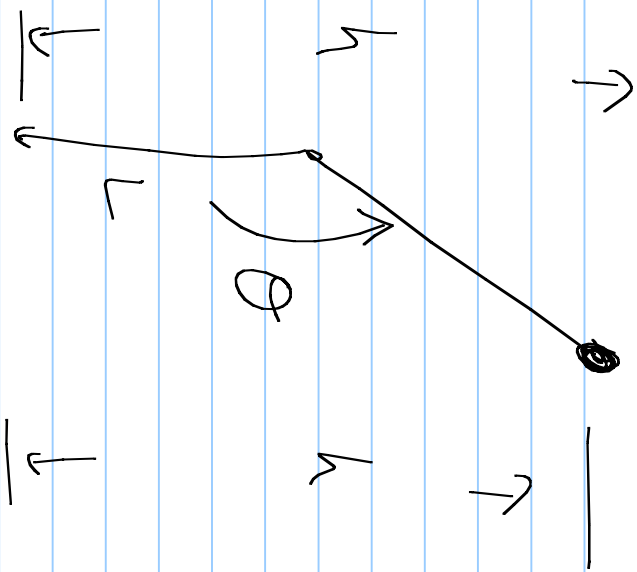
$$\frac{d^2 \theta}{dt^2} = 0 = \frac{4KL^2}{P} (1 - \cos \theta) \sin \theta - mgL \cos \theta$$

Suppose $m = 500g$, $L = 200mm$, $\theta = 45^\circ$ $K = ?$

$$K = \frac{mg \cos \theta}{4L(1 - \cos \theta) \sin \theta} = \frac{(0.5)(9.81)}{(0.8)(1 - \frac{1}{\sqrt{2}})} = 20.9 N/m$$

Stability

$P'(s) = 0$ $P''(s) > 0$ stable < 0 unstable



$$P_y = mgh$$

$$h = L - L \cos \theta \quad \Rightarrow \quad P_y = mgL(1 - \cos \theta)$$

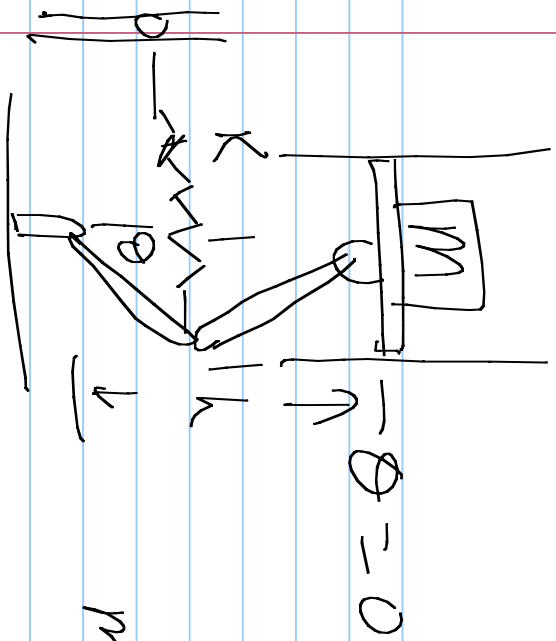
$$\frac{dP_y}{d\theta} = mgL \sin \theta = 0$$

$$\sin \theta = 0$$

$$\theta = 0, \pi$$

$$\frac{d^2 P_y}{d\theta^2} = mgL \cos \theta$$

$$\text{@ } \theta = 0 \quad P_y'' = mgL > 0 \quad \text{@ } \theta = \pi \quad P_y'' = -mgL < 0$$



$$\theta = 0$$

$$h = 2L \cos \theta$$

$$x = L \sin \theta$$

$$mg \cdot 2L \cos \theta + \frac{1}{2} K L^2 \sin^2 \theta = P$$

$$\frac{dP}{d\theta} = 2L^2 \sin \theta \cos \theta - 2mgL \sin \theta = 0$$

$$L \sin \theta [2L \cos \theta - 2mg] = 0$$

$$\sin \theta = 0 \quad \text{or} \quad 2L \cos \theta - 2mg = 0$$

$$\theta = 0, \pi \quad \cos \theta = \frac{2mg}{2L}$$

$$\frac{d^2 P}{d\theta^2} = KL^2 (\cos^2 \theta - \sin^2 \theta) - 2mgl \cos \theta$$

$$\text{@ } \theta = 0, \quad KL^2 - 2mgl > 0$$

$$KL > 2mg$$

$$\text{@ } \theta = \pi, \quad KL^2 + 2mgl > 0$$

$$\text{@ } \cos \theta = \frac{2mg}{KL}$$

$$\frac{d^2 P}{d\theta^2} = KL^2 (2 \cos^2 \theta - 1) - 2mgl \cos \theta$$

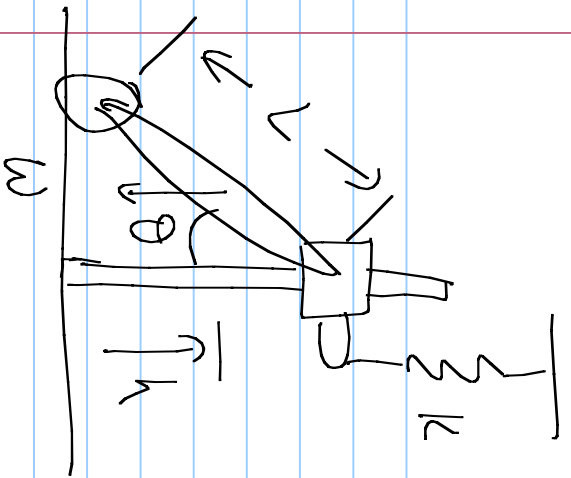
$$kL^2 \left(2 \left[\frac{4m^2 g^2}{k^2 L^2} \right] - 1 \right) - 2mg \frac{2mg}{k}$$

$$= \frac{8m^2 g^2}{k} - kL^2 - \frac{4m^2 g^2}{k} = \frac{4m^2 g^2}{k} - kL^2 > 0$$

$$4m^2 g^2 > kL^2$$

$$2mg > kL$$

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$$\theta = 0$$

$$x = L - L \cos \theta$$

$$h = \frac{L}{2} \cos \theta$$

$$P = \frac{1}{2} R L^2 (1 - \cos \theta)^2 + \frac{m g L}{2} \cos \theta$$

$$P' = R L^2 (1 - \cos \theta) \sin \theta - \frac{m g L}{2} \sin \theta = 0$$

$$\sin \theta = 0 \quad \text{or} \quad R L^2 (1 - \cos \theta) - \frac{m g L}{2} = 0$$

$$P'' = R L^2 (1 - \cos \theta) \cos \theta + R L^2 \sin^2 \theta - \frac{m g L}{2} \cos \theta$$

$$\text{At } \theta = 0 \quad R L^2 - \frac{m g L}{2} > 0 \quad R L > \frac{1}{2} m g$$

$$1 - \cos \theta = \frac{m g}{2kL} \Rightarrow \cos \theta = 1 - \frac{m g}{2kL}$$

$$P'' = \left(kL^2 - \frac{m g^2 L}{2} \right) \cos \theta + kL^2 \left(1 - 2 \cos^2 \theta \right)$$

$$= \left(kL^2 - \frac{m g^2 L}{2} \right) \left[1 - \frac{m g}{2kL} \right] + kL^2 \left[1 - 2 \left(1 - \frac{m g}{2kL} \right)^2 \right]$$

$$= \frac{1}{k} \left[kL - \frac{m g}{2} \right] \left[kL - \frac{m g}{2} \right] + kL^2 \left[1 - 2 + \frac{4m g}{2kL} - \frac{2m^2 g^2}{4k^2 L^2} \right]$$

$$= \frac{1}{k} \left(kL - \frac{m g}{2} \right)^2 - \frac{1}{k} \left[kL^2 - 2m g kL + \frac{m^2 g^2}{2} \right]$$

$$= \frac{1}{R} \left[\frac{KL^2}{4} - mgKL + \frac{m^2 g^2 L}{4} - \frac{KL^2}{4} + 2mgKL - \frac{m^2 g^2 L}{2} \right]$$

$$= \frac{1}{R} \left[mgKL - \frac{m^2 g^2 L}{4} \right] = \frac{m g}{R} \left[KL - \frac{m g}{4} \right] > 0$$

$$KL - \frac{m g}{4} > 0 \Rightarrow KL > \frac{m g}{4}$$

$$mg = 30 \text{ lbs}, \quad L = 3 \text{ ft}, \quad K = 15 \text{ lb/ft} +$$

$$KL = 45 \text{ lbs} \quad \frac{KL}{4} > \frac{m g}{4} \quad \underline{\underline{\theta = 0}}$$