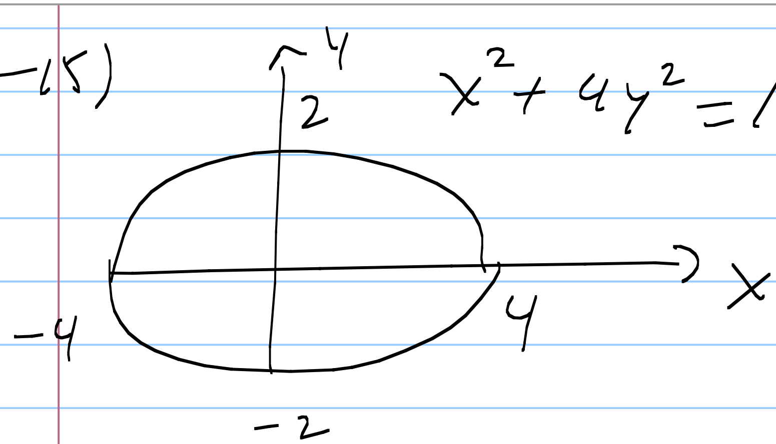
EGR 180 7/23

Note Title 7/23/2009

10-15)



$x^2 + 4y^2 = 16 \Rightarrow y = \sqrt{4 - \frac{x^2}{4}}$

$$I_x = \iint_R y^2\,dA = \int_{-4}^{4}\int_{-\sqrt{4-\frac{x^2}{4}}}^{\sqrt{4-x^2/4}} y^2\,dy\,dx = 4\int_0^4\int_0^{\sqrt{\ }} y^2\,dy\,dx$$

$$= 4\int_0^4 \frac{y^3}{3}\Big|_0^{\sqrt{4-x^2/4}} dx = \frac{4}{3}\int_0^4 \left(4-\frac{x^2}{4}\right)^{3/2} dx$$

$x = 4\sin\theta \qquad dx = 4\cos\theta\, d\theta$

$$I_x = \frac{4}{3}\int_0^{\pi/2} 8\cos^3\theta \cdot 4\cos\theta\, d\theta = \frac{128}{3}\int_0^{\pi/2} \cos^4\theta\, d\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2} \qquad \cos^4\theta = \frac{1+2\cos(2\theta)+\cos^2 2\theta}{4}$$

$$= \frac{1}{4} + \frac{1}{2}\cos(2\theta) + \frac{1}{8}\left[1+\cos(4\theta)\right]$$

$$I_x = \frac{128}{3_8}\int_0^{\pi/2}\left[\frac{3}{8} + \cancel{\frac{1}{2}\cos(2\theta)}^{\,0} + \cancel{\frac{1}{8}\cos(4\theta)}^{\,0}\right] d\theta$$

$$= \frac{12\cancel{8}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{8}}\cdot\frac{\pi}{\cancel{2}} = 8\pi$$

b) $I_y = \iint_R x^2\,dA = \int_{-2}^{2} \int_{-\sqrt{16-4y^2}}^{\sqrt{16-4y^2}} x^2\,dx\,dy$

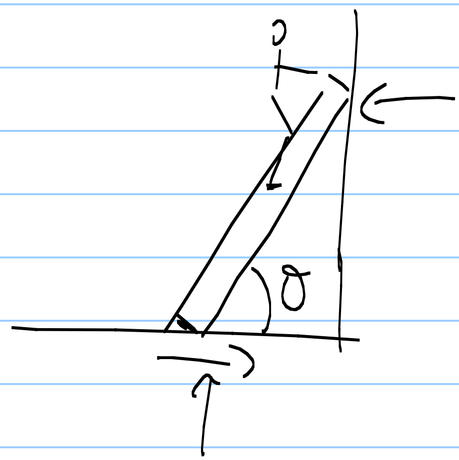
$= \frac{4}{3} \int_0^2 x^3 \Big|_0^{\sqrt{16-4y^2}} dy = \frac{4}{3} \int_0^2 (16-4y^2)^{3/2}\,dy$

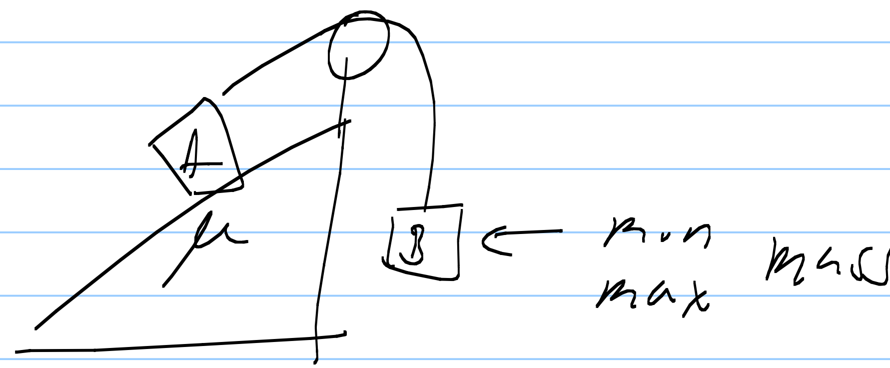
$y = 2\sin\theta$
$dy = 2\cos\theta\,d\theta$

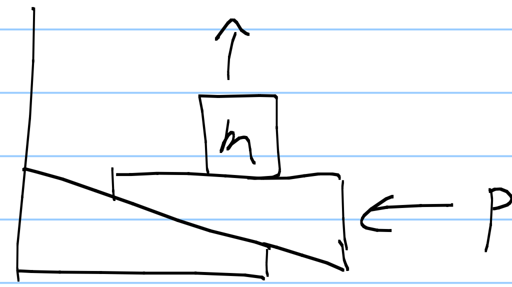
$= \frac{4}{3} \int_0^{\pi/2} 64\cos^3\theta \cdot 2\cos\theta\,d\theta$

$= \frac{512}{3} \int_0^{\pi/2} \cos^4\theta\,d\theta = \frac{\cancel{512}^{32}}{\cancel{3}} \left(\frac{\cancel{3}}{\cancel{8}} \cdot \frac{\pi}{\cancel{2}} \right) = 32\pi$

$$\left[a^2 - b^2 x^2\right]^{1/2} \qquad x = \frac{a}{b}\sin\theta$$





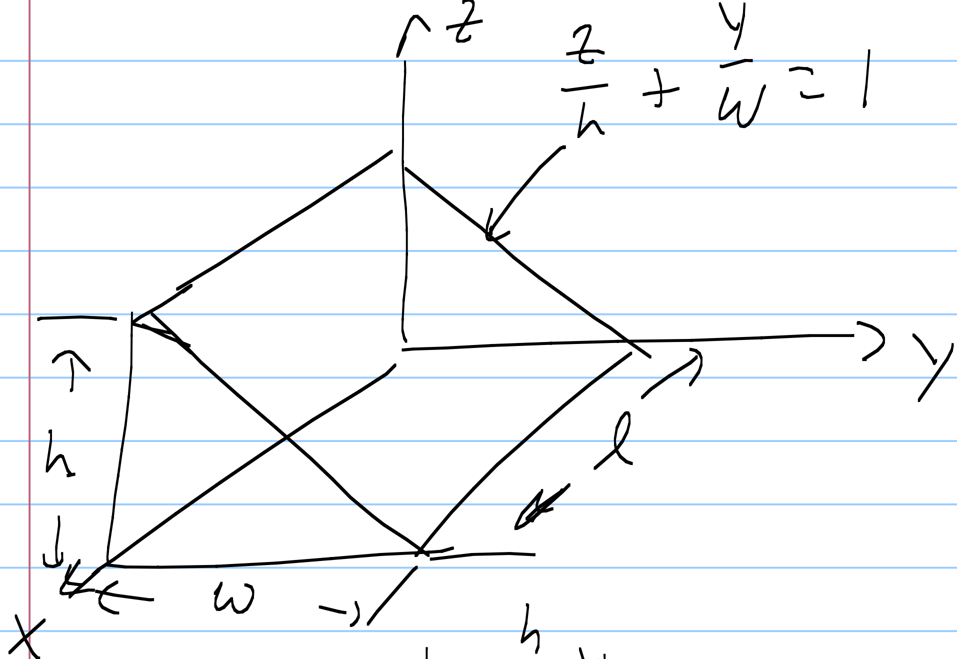


Wedge

Screw Problem

Flat Belt

Journal Bearings



$$\frac{z}{h} + \frac{y}{w} = 1$$

$$I_x = \iiint_V (y^2+z^2)\, dV$$

$$I_y = \iiint_V (x^2+z^2)\, dV$$

$$I_z = \iiint_V (x^2+y^2)\, dV$$

$$I_{xy} = \iiint_V xy\, dV$$

$$I_{xz} = \iiint_V xz\, dV$$

$$I_{yz} = \iiint_V yz\, dV$$

$$I_x = \int_0^l \int_0^w \int_0^{h-\frac{h}{w}y} (y^2+z^2)\, dz\, dy\, dx$$

$$= l \int_0^w \left[y^2 z + \frac{z^3}{3} \right|_0^{h-\frac{h}{w}y} dy$$

$$\frac{h}{3}(w-y)^3$$

$$= l \int_0^{w} h y^2 - \frac{h}{w} y^3 + \frac{1}{3}\left(h - \frac{h}{w} y\right)^3 dy$$

$$= l \left(\frac{h y^3}{3} - \frac{h y^4}{4w} - \frac{h^3}{12 w^3} (w-y)^4 \right) \Big|_0^{w} = l \left[\frac{h w^3}{3} - \frac{h w^3}{4} + \frac{h^3}{12 \cancel{w^3}} \cdot w^{\cancel{4}} \right]$$

$$= \frac{l h w}{12} \left[w^2 + h^2 \right]$$

$$I_y = \int_0^{l} \int_0^{w} \int_0^{h - \frac{h}{w} y} (x^2 + z^2)\, dz\, dy\, dx = \int_0^{l} \int_0^{w} \left[x^2 \left(h - \frac{h}{w} y\right) + \frac{\left(h - \frac{h}{w} y\right)^3}{3} \right] dy\, dx$$

$$= \int_0^l x^2 h y - \frac{x^2 h}{2w} y^2 - \frac{h^3}{12w^3}(w-y)^4 \Big|_0^w dx$$

$$= \int_0^l \left(h w x^2 - \frac{hw}{2} x^2 + \frac{h^3 w}{12}\right) dx = \frac{h w x^3}{6} + \frac{h^3 w}{12} x \Big|_0^l$$

$$= \frac{h w l^3}{6} + \frac{h^3 w l}{12}$$

$$I_z = \int_0^l \int_0^w \int_0^{h - \frac{h}{w} y} (x^2 + y^2)\, dz\, dy\, dx = \int_0^l \int_0^w x^2\left(h - \frac{h}{w} y\right) + h y^2 - \frac{h}{w} y^3 \, dy\, dx$$

$$= \int_0^l x^2 h y - \frac{h x^2}{2w} y^2 + \frac{h y^3}{3} - \frac{h y^4}{4w} \Big|_0^w dx$$

$$= \int_0^{\ell} \left[hwx^2 - \frac{hw}{2}x^2 + \frac{hw^3}{3} - \frac{hw^3}{4} \right] dx$$

$$= \int_0^{\ell} \left(\frac{hw}{2}x^2 + \frac{hw^3}{12} \right) dx = \frac{hwx^3}{6} + \frac{hw^3}{12}x \Big|_0^{\ell}$$

$$= \frac{hw\ell^3}{6} + \frac{hw^3\ell}{12}$$

$$I_{xy} = \iiint_V xy\,dV = \int_0^{\ell} \int_0^{w} \int_0^{h - \frac{h}{w}y} xy\,dz\,dy\,dx$$

$$= \frac{\ell^2}{2} \int_0^{w} y\left(h - \frac{h}{w}y\right) dy = \frac{\ell^2}{2} \left| \frac{hy^2}{2} - \frac{hy^3}{3w} \right|_0^{w}$$

$$= \frac{l^2}{2}\left(\frac{hw^2}{2} - \frac{hw^2}{3}\right) = \frac{hw^2l^2}{12}$$

$$I_{yz} = \int_0^l \int_0^w \int_0^{h-\frac{h}{w}y} yz \, dz \, dy \, dx$$

$$= \frac{l}{2}\int_0^w y\left(h - \frac{h}{w}y\right)^2 dy$$

$$= \frac{l}{2}\int_0^w \left(h^2 y - \frac{2h^2}{w}y^2 + \frac{h^2}{w^2}y^3\right) dy$$

$$= \frac{l}{2}\left[\frac{\overset{6}{h^2w^2}}{2} - \frac{\overset{8}{2h^2w^2}}{3} + \frac{\overset{3}{h^2w^2}}{4}\right] = \frac{lh^2w^2}{24}$$

$$\frac{z}{c} + \frac{x}{a} + \frac{y}{b} = 1$$

$$\int_0^a \int_0^{b(1-\frac{x}{a})} \int_0^{c - \frac{c}{a}x - \frac{c}{b}y}$$

$$I_{xz} = \int_0^{\ell} \int_0^{w} \int_0^{h - \frac{h}{w} y} xz \, dz \, dy \, dx = \frac{\ell^2}{4} \int_0^{w} \left(h - \frac{h}{w} y\right)^2 dy$$

$$u = h - \frac{h}{w} y$$

$$du = -\frac{h}{w} dy$$

$$= \frac{\ell^2}{4} \int_0^{h} u^2 \, \frac{w}{h} \, du$$

$$= \frac{\ell^2 w}{12 h} u^3 \Big|_0^{h} = \frac{w \ell^2 h^2}{12}$$