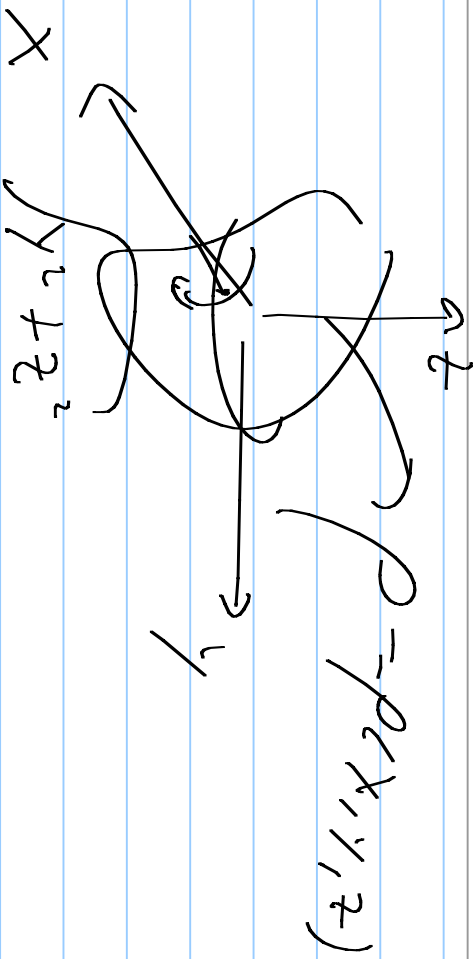


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$$I_x = \iint_R y^2 dA$$



$$I_x = \iiint_V (y^2 + z^2) dm \quad dm = \rho dV$$

$$K_x = \sqrt{\frac{I_x}{m}}$$

$$I_y = \iiint_V (x^2 + z^2) dm$$

$$K_y = \sqrt{\frac{I_y}{m}}$$

$$I_z = \iiint_V (x^2 + y^2) dm$$

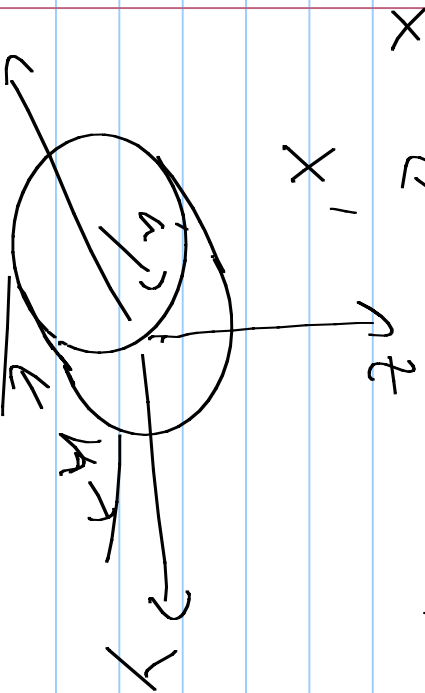
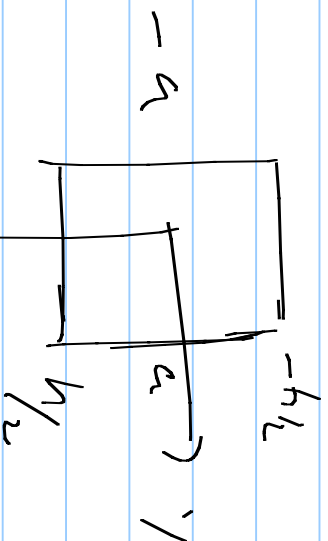
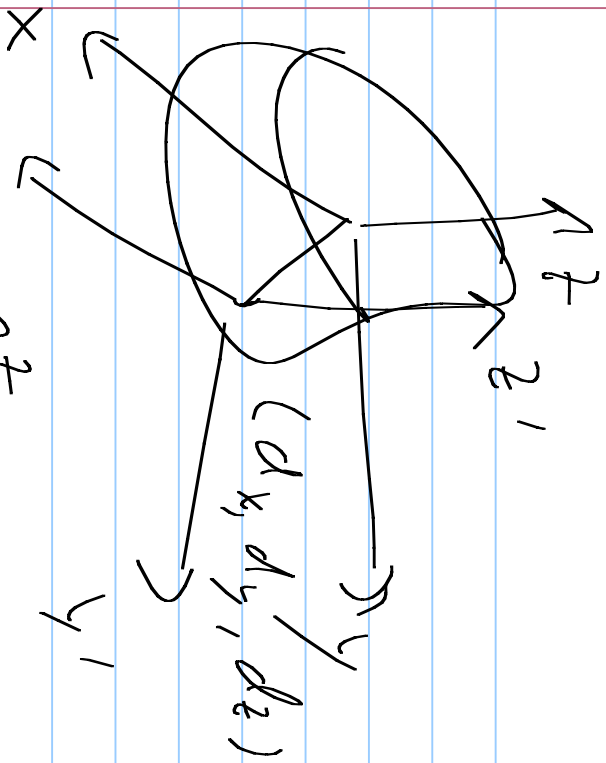
$$K_z = \sqrt{\frac{I_z}{m}}$$

Center of mass

$$I_{x'} = I_x + (d_y^2 + d_z^2) m$$

$$I_{y'} = I_y + (d_x^2 + d_z^2) m$$

$$I_{z'} = I_z + (d_x^2 + d_y^2) m$$



$$\rho(x, y, z) = \frac{\rho_0}{h^2} (x^2 + h^2)$$

$$I_x = \iiint_V (y^2 + z^2) \, dV = \int_{-h/2}^{h/2} \int_0^a \int_0^{2\pi} r^2 \frac{\rho_0}{h^2} (x^2 + y^2) \, r \, dr \, d\theta \, dz$$

$$= \frac{2\pi\rho_0}{h^2} \int_{-h/2}^{h/2} \int_0^a r^3 \, dr (x^2 + y^2) \, dx$$

$$= \frac{2\pi\rho_0}{h^2} \left(\frac{x^3}{3} + h^2 x \right) \Big|_{-h/2}^{h/2} \left(\frac{r^4}{4} \right) \Big|_0^a$$

$$= \frac{2\pi\rho_0}{h^2} \left(\frac{h^3}{24} + \frac{h^3}{2} \right) \left(\frac{a^4}{4} \right) = \frac{13\pi\rho_0 h a^4}{24}$$

$$M = \iiint_V dm = \int_{-h/2}^{h/2} \int_0^{h^2} \int_0^{2\pi} \rho_0 \left(x^2 + h^2 \right) r dr dx$$

$$= \frac{2\pi\rho_0}{h^2} \int_{-h/2}^{h/2} (x^2 + h^2) dx \cdot \int_0^4 r dr$$

$$= \frac{2\pi\rho_0}{h^2} \left[\frac{x^3}{3} + h^2x \right]_{-h/2}^{h/2} \left[\frac{r^2}{2} \right]_0^4$$

$$= \frac{2\pi\rho_0}{h^2} \left[\frac{h^3}{24} + \frac{h^3}{2} \right] \frac{a^2}{2} = \frac{13\pi\rho_0 h a^2}{12}$$

$$a^2$$

$$K_x^2 = \frac{I_x}{m} = \frac{137 \pi a^4 h}{24} = \frac{a^2}{2}$$

$$\frac{137 \pi a^4 h}{24} = \frac{a^2}{2}$$

$$K_x = \frac{a}{\sqrt{2}}$$

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$I_y = \iiint_V (x^2 + z^2) dm = \int_{-h/2}^{h/2} \int_0^a \int_0^{2\pi} [x^2 + r^2 \sin^2 \theta]$$

$$\frac{\rho}{h^2} (x^2 + h^2) r dr d\theta dx$$

$$\frac{2\rho a^4 h}{h^2} \int_{-h/2}^{h/2} \int_0^a (x^2 + \frac{r^2}{2}) (x^2 + h^2) r dr dx$$

$$= \frac{2\pi\rho_0}{h^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} (x^2 + h^2) \int_0^4 (x^2 r + \frac{r^3}{2}) dr dx$$

$$= \frac{2\pi\rho_0}{h^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} (x^2 + h^2) \left[\frac{x^2 r^2}{2} + \frac{r^4}{8} \right]_0^4 dx$$

$$= \frac{2\pi\rho_0}{h^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} (x^2 + h^2) \left[\frac{a^4}{8} + \frac{a^2 x^2}{2} \right] dx$$

$$= \frac{\pi \rho_0 a^2}{4h^2} \int_{-h/2}^{h/2} (x^2 + h^2) (a^2 + 4x^2) dx$$

$$= \frac{\pi \rho_0 a^2}{2h^2} \int_0^{h/2} (4x^4 + (a^2 + 4h^2)x^2 + a^2h^2) dx$$

$$= \frac{\pi \rho_0 a^2}{2h^2} \left[\frac{4}{5} x^5 + \frac{(a^2 + 4h^2)}{3} x^3 + a^2 h^2 x \right]_0^{h/2}$$

$$= \frac{\pi \rho_0 a^2}{2h^2} \left[\frac{4}{5} \cdot \frac{h^5}{32} + \frac{(a^2 + 4h^2)h^3}{24} + \frac{a^2 h^3}{2} \right]$$

$$= \frac{\pi \rho_0 a^2}{2} \left[\frac{h^3}{40} + \frac{a^2 h + 4h^3}{24} + \frac{a^2 h}{2} \right]$$

$$= \frac{\pi \rho_0 a^2}{2} \left[\frac{3h^3 + 5a^2 h + 20h^3 + 10a^2 h}{120} \right]$$

$$= \frac{\pi \rho_0 a^2}{240} \left[23h^3 + 65a^2 h \right]$$

$$k_y^2 = \frac{I_y}{m} = \frac{\cancel{\pi \rho_0 a^2} (23h^3 + 65a^2 h)}{240}$$

$$\frac{13 \cancel{\pi \rho_0 a^2} h}{12}$$

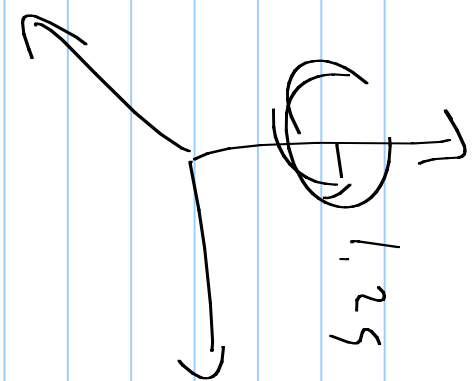
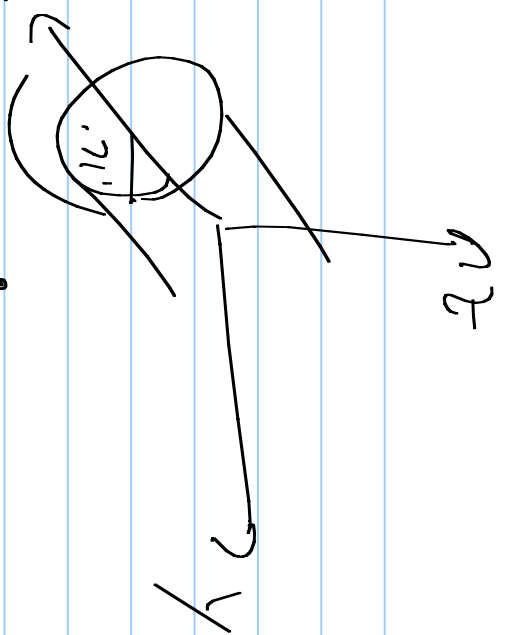
$$K_y = \frac{(23h^2 + 65a^2)}{260} \cdot K_y = \sqrt{\frac{23h^2 + 65a^2}{260}}$$

$$a = 1 \text{ ft}$$

$$h = 4 \text{ ft}$$

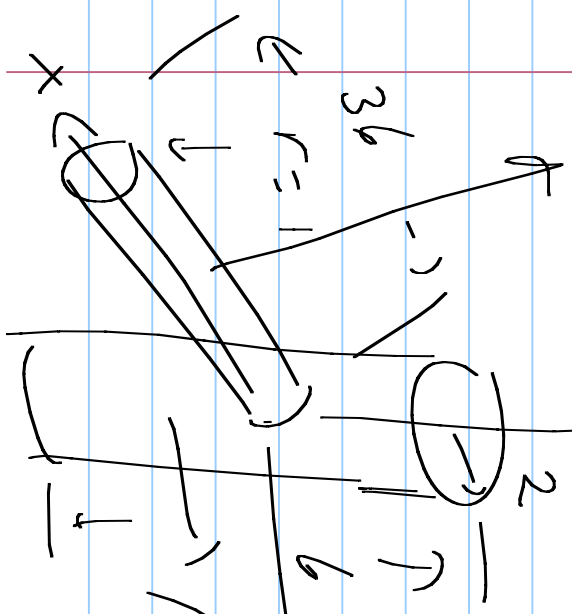
$$K_x = \frac{a}{\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ ft} = 0.71 \text{ ft} \leftarrow$$

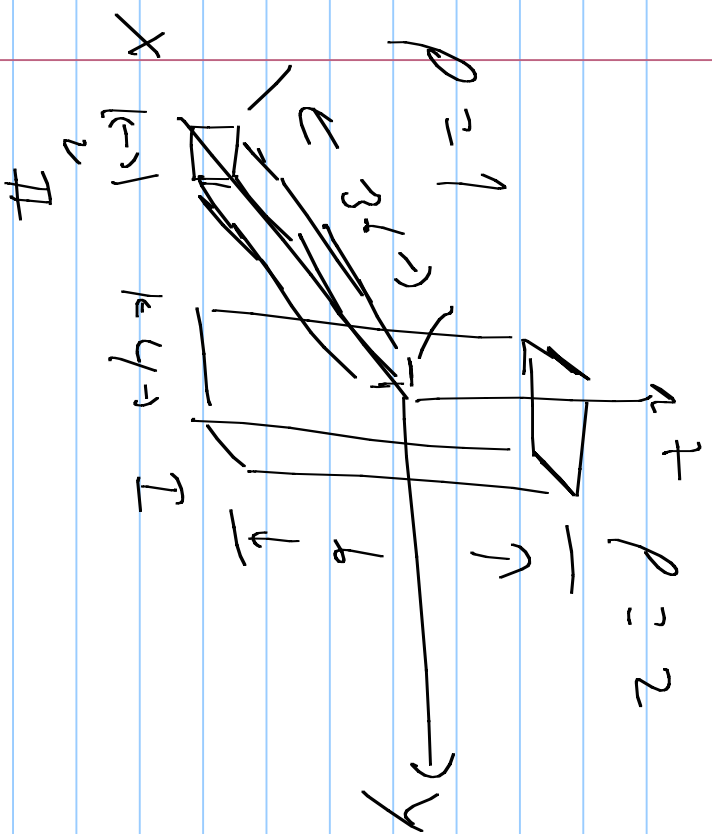
$$K_y = K_z = \left[\frac{23 \cdot 16 + 65 \cdot 1}{260} \right]^{1/2} = 1.29 \text{ ft}$$



$r = 1.16 / ft$

$r = 2.14 / ft$





$$I_{xI} = \int_{-2}^2 \int_{-2}^2 \int_0^3 (y^2 + z^2) 2 dz dy dx$$

$$= \int_0^3 \int_{-2}^2 (y^2 z + \frac{z^3}{3}) \Big|_0^3 dy$$

$$= 32 \int_1^2 (3y^2 + 9) dy$$

$$= 32 \int_1^2 (y^3 + 9y) dy$$

$$= 32 [8 + 18] = 832$$

$$I_{yI} = \int_{-2}^2 \int_{-2}^2 \int_{-3}^3 (x^2 + z^2) \cdot 2 \, dz \, dy \, dx = 32 \int_0^2 \int_0^2 \int_0^3 (x^2 + z^2) \, dz \, dy$$

$$= 32 \int_0^2 \int_0^2 \left(x^2 z + \frac{z^3}{3} \right) \Big|_0^3 \, dy \, dx = 32 \int_0^2 \int_0^2 (3x^2 + 9) \, dy \, dx$$

$$= 32 \int_0^2 (x^3 + 9x) \Big|_0^2 \, dx = 832$$

$$I_{zy} = \int_{-2}^2 \int_{-2}^2 \int_{-3}^3 (x^2 + y^2) \cdot 2 \, dz \, dy \, dx$$

$$\begin{aligned}
 &= 48 \int_0^2 \int_0^2 (x^2 + y^2) dy dx \\
 &= 48 \int_0^2 \left(x^2 y + \frac{y^3}{3} \right) \Big|_0^2 dx = 48 \int_0^2 \left(2x^2 + \frac{8}{3} \right) dx
 \end{aligned}$$

$$= 48 \left(\frac{2}{3} x^3 + \frac{8}{3} x \right) \Big|_0^2 = 48 \left(\frac{16}{3} + \frac{16}{3} \right) = 512$$

$$\begin{aligned}
 I_{XII} &= \int_{-2}^2 \int_{-1}^1 \int_{-1}^1 (y^2 + z^2) dz dy dx = 144 \int_0^1 \int_0^1 (y^2 + z^2) dz dy \\
 &= 144 \int_0^1 \left(y^2 z + \frac{z^3}{3} \right) \Big|_0^1 dy = 144 \int_0^1 \left(y^2 + \frac{1}{3} \right) dy
 \end{aligned}$$

$$= 144 \left(\frac{1}{5} + \frac{1}{5} \right) = 96$$

$$I_{yII} = \int_2^{38} \int_{-1}^1 \int_{-1}^1 (x^2 + z^2) dz dy dx$$

$$= 4 \int_2^{38} x^2 z + \frac{z^3}{3} \Big|_{-1}^1 dx = 4 \int_2^{38} \left(x^2 + \frac{1}{3} \right) dx$$

$$= 4 \left[\frac{x^3}{3} + \frac{x}{3} \right]_2^{38} = 73,200 = I_{zII}$$

$$I_x = I_{xI} + I_{xII} = 832 + 96 = 928$$

$$I_Y = I_{Y_I} + I_{Y_{II}} = 8322 + 73,200 = 74,032$$

$$I_Z = I_{Z_I} + I_{Z_{II}} = 512 + 23,200 = 23,712$$