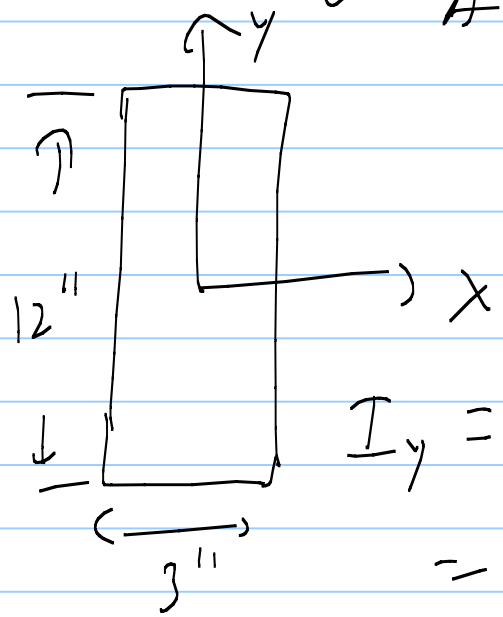
EGR 180 7/21

Note Title 7/21/2009

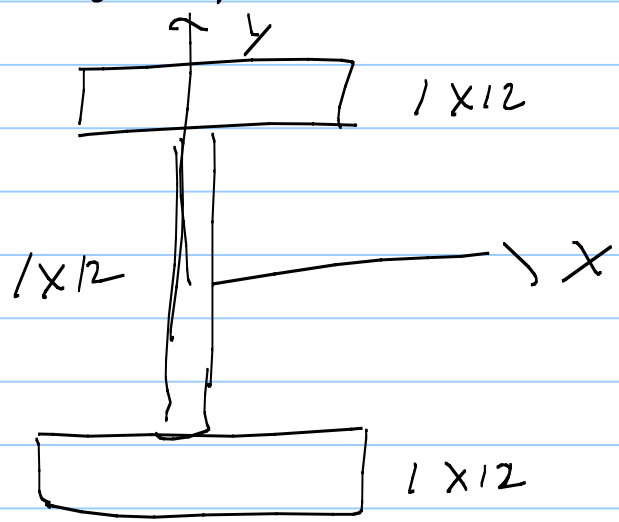
Radius of Gyration

$$K_x = \sqrt{\frac{I_x}{A}} \qquad K_y = \sqrt{\frac{I_y}{A}}$$



$$I_y = \frac{(3)^3(12)}{12}$$

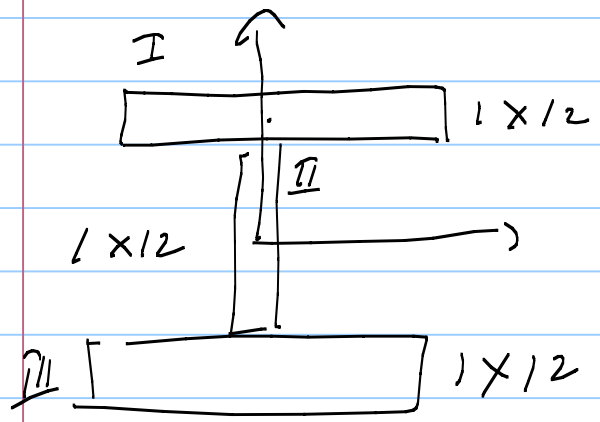
$$= 27 \text{ in}^4$$



$$I_x = \frac{(12)^3}{12} = 432 \text{ in}^4$$

$$K_x = \sqrt{\frac{432}{36}} = \sqrt{12} = 3.464 \text{ in}$$

$$K_y = \sqrt{\frac{27}{36}} = 0.866$$



$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3}$$

$$I_{x_1} = \frac{(1)^3(12)}{12} + (6.5)^2 \cdot 12 \qquad \frac{a^3 b}{12}$$

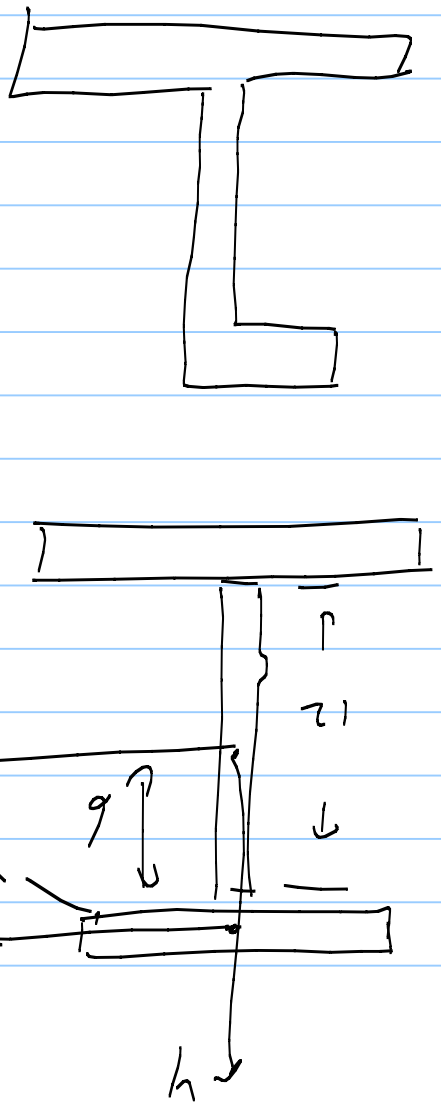
$$= 1 + 507 = 508$$

$$I_{x_2} = \frac{(12)^3(1)}{12} = 144 \qquad I_{x_3} = 508$$

$$I_x = 1160 \qquad K_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1160}{36}} = \underline{5.68}$$

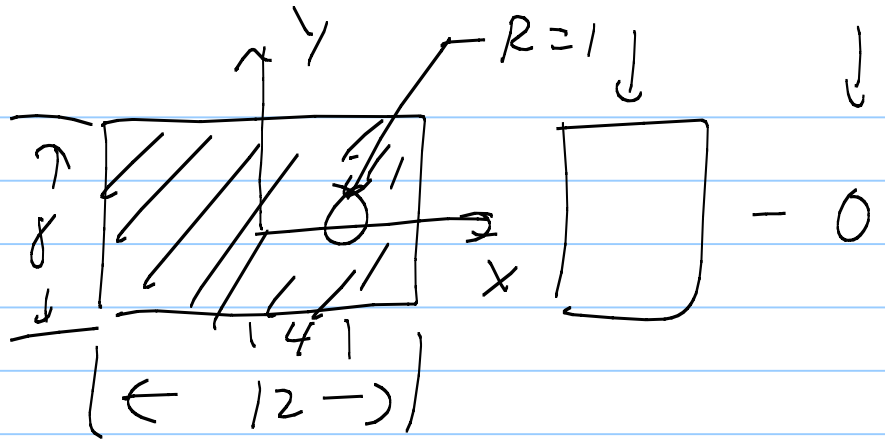
$$I_{y_1} = \frac{(12)^3(1)}{12} = 144 = I_{y_3} \qquad I_y = 289$$

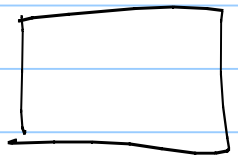
$$I_{y_2} = \frac{(1)^3(12)}{12} = 1 \qquad K_y = \sqrt{I_y/A} = \underline{2.83}$$



$$I_x = \sum I_{x_i}$$

$$I_y = \sum I_{y_i}$$





$$I_x = \frac{(8)^3(12)}{12} = 512$$

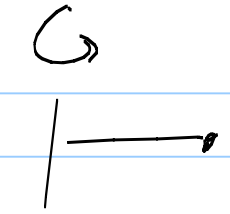
$$I_y = \frac{(12)^3(8)}{12} = 1152$$

$$0 = I_x = \frac{\pi R^4}{4} + 0 = \frac{\pi}{4}$$

$$I_y = \frac{\pi R^4}{4} + (4)^2(\pi) = 16.25\,\pi$$

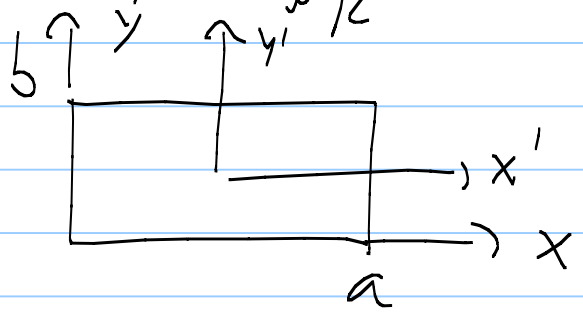
$$I_x = 512 - \pi/4 = 511.2$$

$$I_y = 1152 - 16.25\pi = 1100.9$$



Mixed Second Moments

$$I_{xy} = \iint_R xy\,dA$$



$$I_{xy} = \int_0^a \int_0^b xy\,dy\,dx = \frac{a^2}{2} \cdot \frac{b^2}{2} = \frac{a^2 b^2}{4}$$

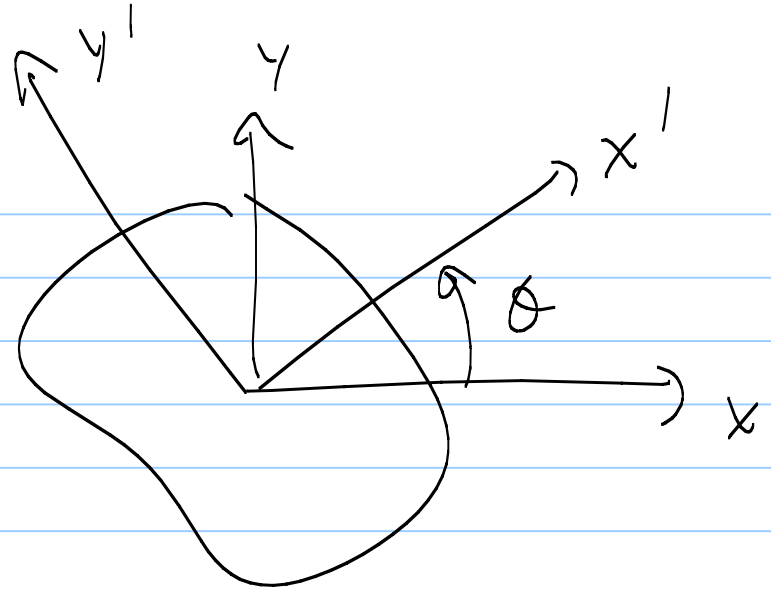
$$I_{x'y'} = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} xy\,dx\,dy = 0$$

$$I_{x'y'} = \iint_R (x+d_x)(y+d_y)\,dA = \iint_R (xy + xd_y + yd_y + d_x d_y)\,dA$$

$$= I_{xy} + d_y \iint_R y\,dA + d_x \iint_R x\,dA + d_x d_y \iint_R dA$$

$$= I_{xy} + d_x d_y A$$

$$I_{x'y'} = \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) ab = \frac{a^2 b^2}{4}$$



$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$I_{x'} = \iint_R (y')^2 \, dA = \iint_R \left[x\cos\theta - x\sin\theta \right]^2 dA$$

$$= \iint_R \left(y^2\cos^2\theta - 2xy\cos\theta\sin\theta + x^2\sin^2\theta \right) dA$$

$$= I_x\cos^2\theta - 2\cos\theta\sin\theta\, I_{xy} + \sin^2\theta\, I_y$$

$$I_{x'y'} = \iint_R x'y'\,dA = \iint_R (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)\,dA$$

$$= -\cos\theta\sin\theta\, I_y + \cos\theta\sin\theta\, I_x + (\cos^2\theta - \sin^2\theta) I_{xy}$$

$$\frac{d}{d\theta} I_{x'} = -2\cos\theta\sin\theta\, I_x + (2\sin^2\theta - 2\cos^2\theta) I_{xy}$$

$$+ 2\sin\theta\cos\theta\, I_y = 0$$

$$(I_y - I_x)\sin(2\theta) - 2\cos(2\theta) I_{xy} = 0$$

$$\tan(2\theta) = \frac{2 I_{xy}}{I_y - I_x}$$

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin(2\theta) + I_{xy}\cos(2\theta)$$

$$= \frac{\cos(2\theta)}{2}\left[2I_{xy} - (I_y - I_x)\tan(2\theta)\right] = 0$$

$$\Rightarrow I_{y'} = \iint_R (x\cos\theta + y\sin\theta)^2 \, dA = I_y\cos^2\theta + I_x\sin^2\theta + 2\sin\theta\cos\theta\, I_{xy}$$

$$\Rightarrow I_{x'} = I_x\cos^2\theta + I_y\sin^2\theta - 2\sin\theta\cos\theta\, I_{xy}$$

$$\boxed{I_{x'} + I_{y'} = I_x + I_y}$$

$$I_{x'} - I_{y'} = I_x\cos(2\theta) - I_y\cos(2\theta) - 2\sin(2\theta)I_{xy}$$

$$= (I_x - I_y)\cos(2\theta) - 2\sin(2\theta) I_{xy}$$

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin(2\theta) + \cos(2\theta) I_{xy}$$

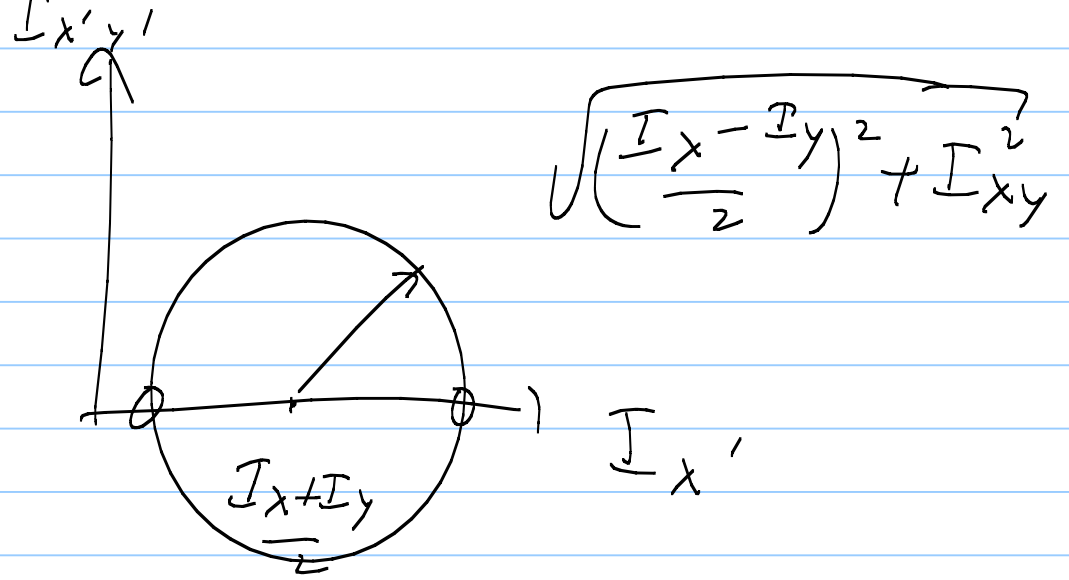
$$\frac{I_{x'} - I_{y'}}{2} = \frac{1}{2}(I_x - I_y)\cos(2\theta) - \sin(2\theta) I_{xy}$$

$$\left(\frac{I_{x'} - I_{y'}}{2}\right)^2 + (I_{x'y'})^2 = \frac{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}{R^2}$$

$$I_{x'} + I_{y'} = I_x + I_y$$

$$I_{y'} = I_x + I_y - I_{x'}$$

$$\left(I_{x'} - \frac{I_x + I_y}{2}\right)^2 + I_{x'y'}^2 = \left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2$$



No Class July 28