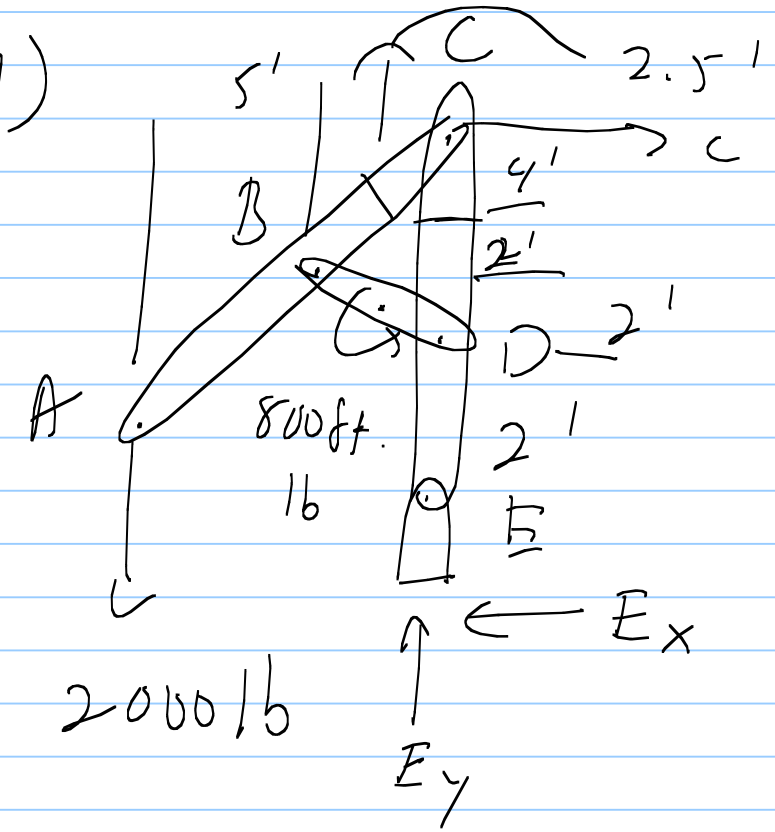
EGR180 7/13

8-31)



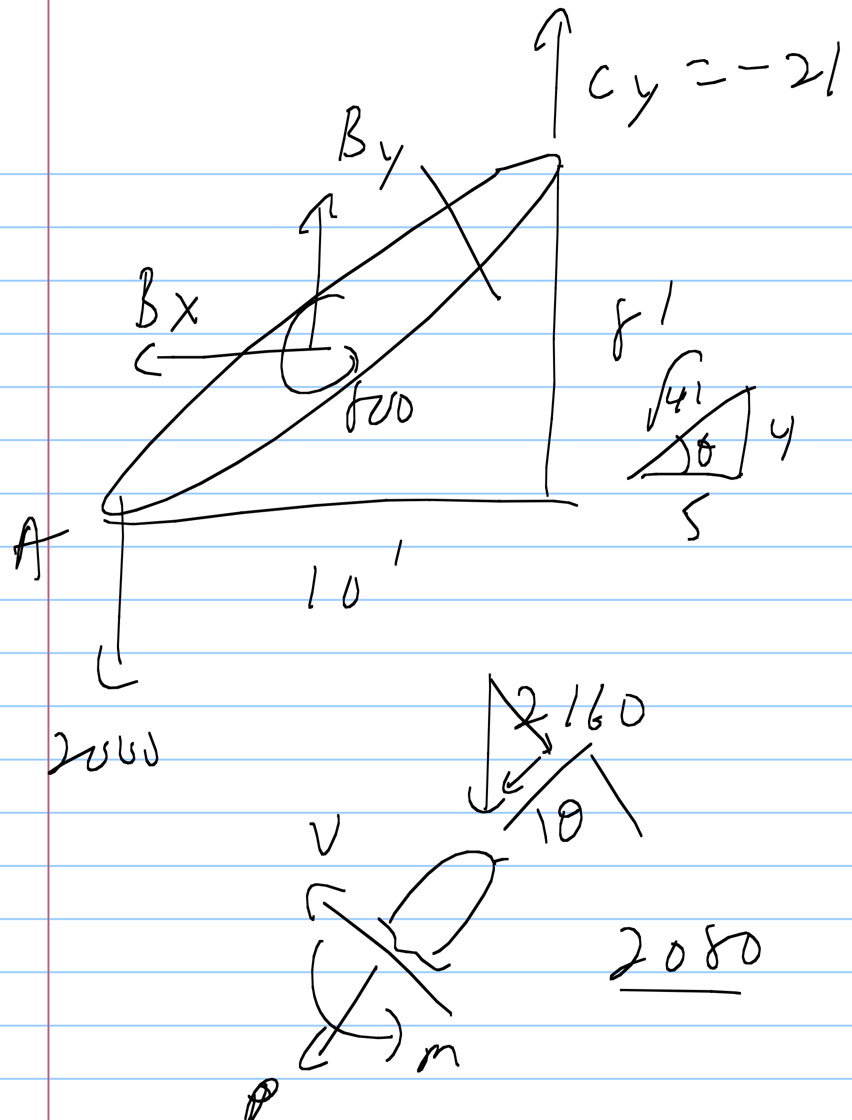
$$\sum F_y \Rightarrow E_y = 2000 \text{ lbs}$$

$$\sum F_x \Rightarrow E_x = C$$

$$\sum M_E = -10 \cdot C + 10 \cdot 2000 + 800 = 0$$

$$10C = 20{,}800$$

$$C = 2080 \text{ lbs}$$



$$\sum M_B = +5 \cdot 2000 + 5C_y + 800 = 0$$

$$C_y = -2160 \text{ lbs}$$

$$\sum F_y = -2000 - 2160 + B_y = 0$$

$$B_y = 4160 \text{ lbs}$$

$$\sum F_x \Rightarrow B_x = 2880 \text{ lbs}$$

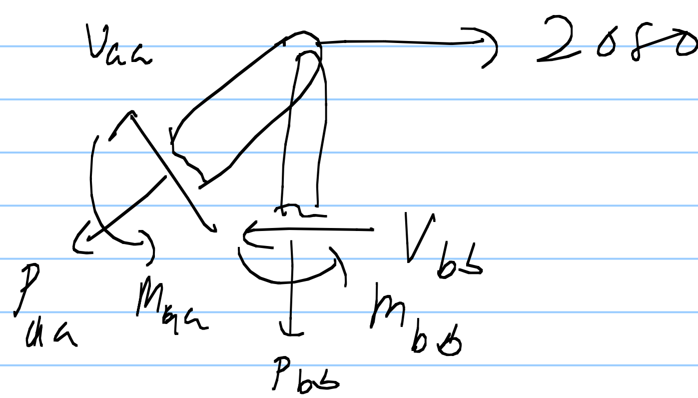
$$\sum F_A = P - 2880 \frac{5}{\sqrt{41}} = 0$$

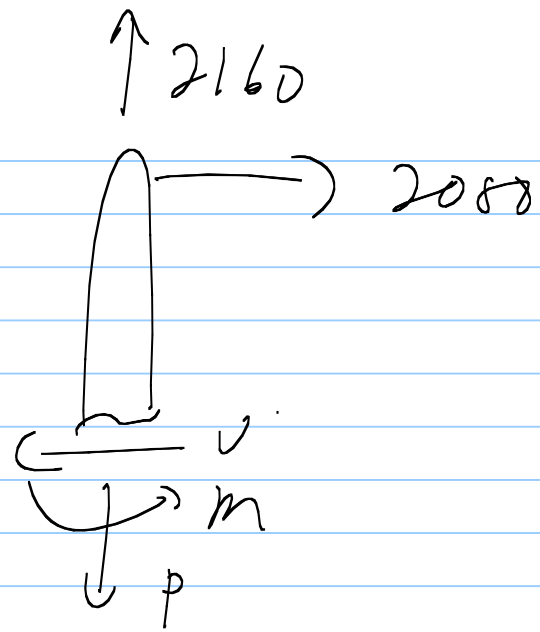
$$P = 1624 \text{ lbs}$$

$$\sum F_v = V - 2160 \cdot \frac{5}{\sqrt{41}} = 0$$

$$V = 1686.7 \text{ lbs}$$

$$\sum M_c = -\frac{\sqrt{41}}{2} V + M = 0 \Rightarrow M = \frac{V\sqrt{41}}{2} = 5400 \text{ ft} \cdot \text{lbs}$$





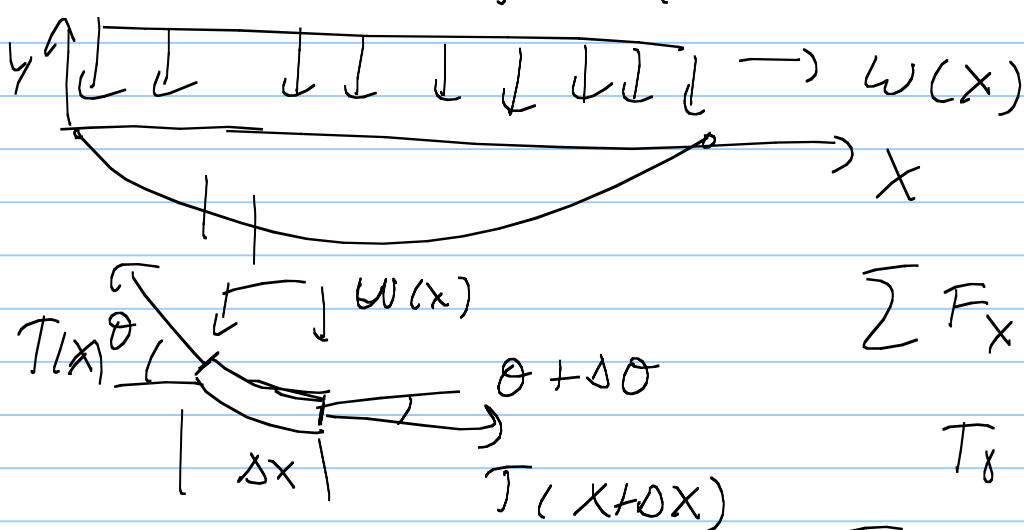
$V = 2080\ lb_f$

$P = 2160\ lb_f$

$M = 4 \cdot V = 8320\ ft\text{-}lb_f$

Case I uniform horizontal loading

Case II uniform density



$$\sum F_x = -T\cos\theta + T(x+\Delta x)\cos(\theta+\Delta\theta) = 0$$

$T_x = T\cos\theta$ is constant

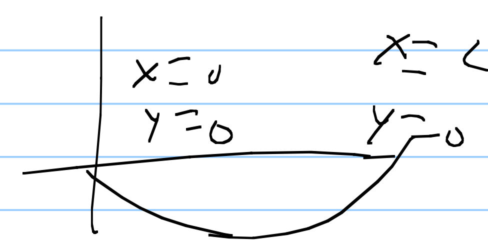
$$\sum F_y = T(x)\sin\theta - T(x+\Delta x)\sin(\theta+\Delta\theta) - w(x)\Delta x = 0$$

$$\frac{T_0}{\cos\theta} = T \Rightarrow \quad T_0 \tan\theta - T_0 \tan(\theta + \Delta\theta) = w(x)\Delta x$$

$$T_0 \left(\frac{\tan(\theta + \Delta\theta) - \tan(\theta)}{\Delta x} \right) = -w(x)$$

$$\tan(\theta) = y' \qquad T_0\, y'' = -w(x) = -w_0$$

$$y'(x) = -\frac{w_0}{T_0} x + c_1$$



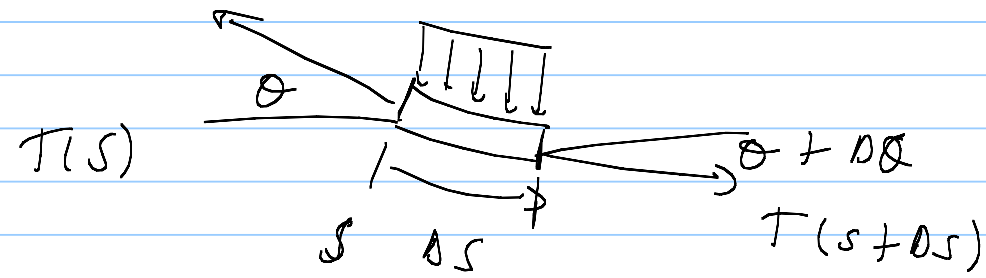
$$y(x) = -\frac{w_0 x^2}{2T_0} + c_1 x + \cancel{c_2}^{\,0}$$

$$y(L) = 0 = -\frac{wL^2}{2T_0} + c_1 L = 0 \Rightarrow c_1 = \frac{wL}{2T_0}$$

$$y(x) = \frac{w}{2T_0} x(L - x)$$

$$y' = \frac{w}{2T_0} [L - 2x) = 0 \Rightarrow x = \frac{L}{2}$$

$$y\left(\frac{L}{2}\right) = \frac{wL^2}{8T_0} = h_{max}$$



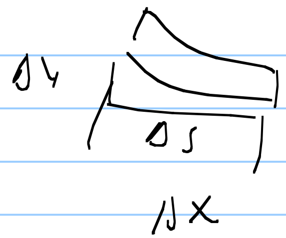
$$\sum F_x = -T(s)\cos\theta + T(s+\Delta s)\cos(\theta+\Delta\theta) = 0$$

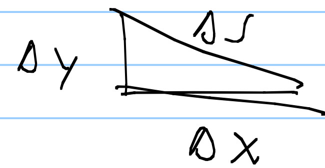
$$T_0 = T(s)\cos(\theta) = \text{const}$$

$$\sum F_y = T(s)\sin(\theta) - T(s+\Delta s)\sin(\theta+\Delta\theta) - w(s)\Delta s = 0$$

$$\Rightarrow T_0\tan(\theta) - T_0\tan(\theta+\Delta\theta) = w(s)\,\Delta s$$

$$T_0\,\frac{\tan(\theta+\Delta\theta) - \tan(\theta)}{\Delta s} = -w(s) = -w_0$$





$= \Delta s^2 = \Delta x^2 + \Delta y^2$

$\Delta s = \sqrt{1+y'^2}\,\Delta x$

$T_0\, y'' = -\omega_0 \sqrt{1+(y')^2}$

$u = y'$

$u' = y''$

$\frac{u'}{\sqrt{1+u^2}} = -\frac{\omega_0}{T_0}$

$\ln\left(u+\sqrt{1+u^2}\right) = -\frac{\omega_0}{T_0}x$

$T_0 \cancel{\cosh(t)}\, t' = -\omega_0 \cancel{\cosh(t)}$

$u = \sinh(t)$

$u' = \cosh(t)\, t'$

$t = -\frac{\omega_0}{T_0}x$

$\frac{dt}{dx} = -\frac{\omega_0}{T_0}$

$$y'(x) = -\sinh\left(\frac{w_0}{T_0}x\right)$$

$$\boxed{y(x) = -\frac{T_0}{w_0}\underline{\underline{\cosh}}\left(\frac{w_0}{T_0}x\right) + C}$$

$$y(x) = \frac{T_0}{w_0}\left[1 - \cosh\left(\frac{w_0}{T_0}x\right)\right]$$