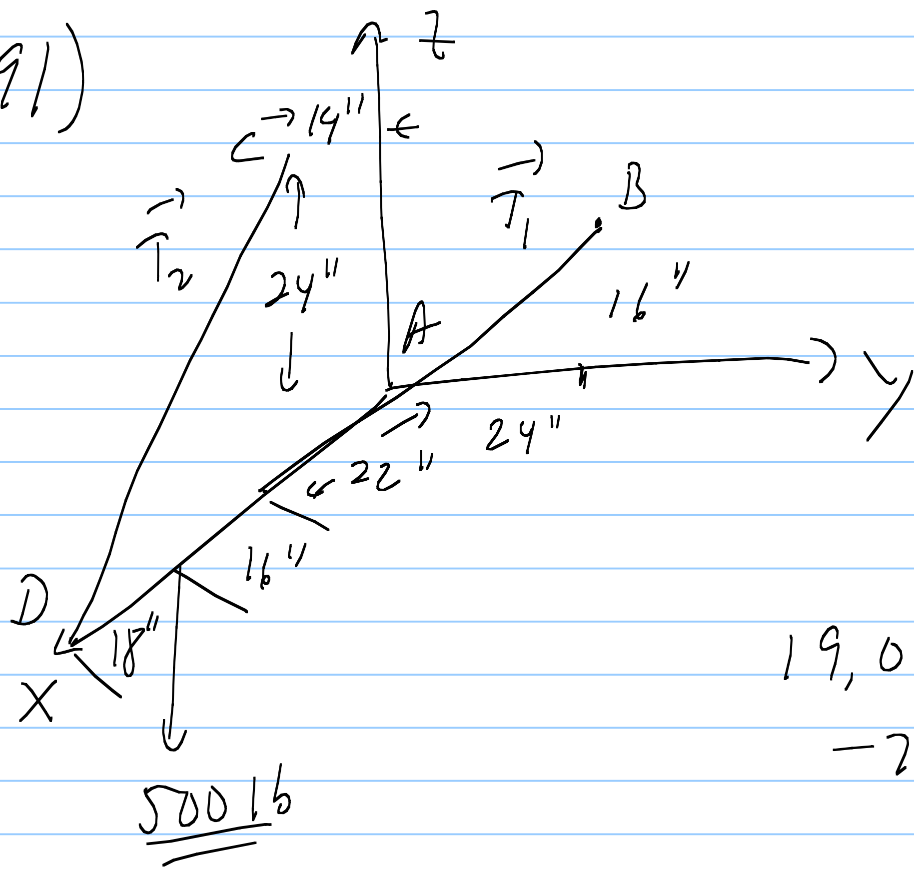
EGR180 7/2

Note Title 7/2/2009

6-91)



$$\vec{T}_1 = R_1(-22\hat{i} + 24\hat{j} + 16\hat{k})$$

$$\vec{T}_2 = R_2(-56\hat{i} - 14\hat{j} + 24\hat{k})$$

$$\sum \vec{M}_A = 38\hat{i} \times -500\hat{k}$$

$$22\hat{i} \times \vec{T}_1 + 56\hat{i} \times \vec{T}_2 = 0$$

$$19{,}000\hat{j} + 528 R_1 \hat{k} - 352 R_1 \hat{j}$$

$$-784 R_2 \hat{k} - 1344 R_2 \hat{j} = 0$$

$\hat{j}: \ 19{,}000 - 352R_1 - 1344R_2 = 0$

$\hat{k}: \ 528R_1 - 284R_2 = 0 \Rightarrow R_1 = \frac{284}{528}R_2 = \frac{49}{33}R_2$

$19{,}000 = \left(352\left(\frac{49}{33}\right) + 1344\right)R_2$

$R_2 = 10.18 \text{ lbs} \qquad R_1 = 15.11 \text{ lbs}$

$\sum F_x = A_x - 22R_1 - 56R_2 = 0 \Rightarrow A_x = 22R_1 + 56R_2 \doteq 902.5 \text{ lbs}$

$\sum F_y = A_y - 14R_2 + 24R_1 = 0 \quad A_y = 14R_2 - 24R_1 = -220.2 \text{ lbs}$

$\sum F_z = A_z + 16R_1 + 24R_2 - 500 = 0 \quad A_z = 13.9 \text{ lbs}$

$$T_1 = R_1\left[22^2 + 24^2 + 16^2\right]^{1/2} = 548.3 \text{ lbs}$$

$$T_2 = R_2\left[56^2 + 14^2 + 24^2\right]^{1/2} = 636.3 \text{ lbs}$$

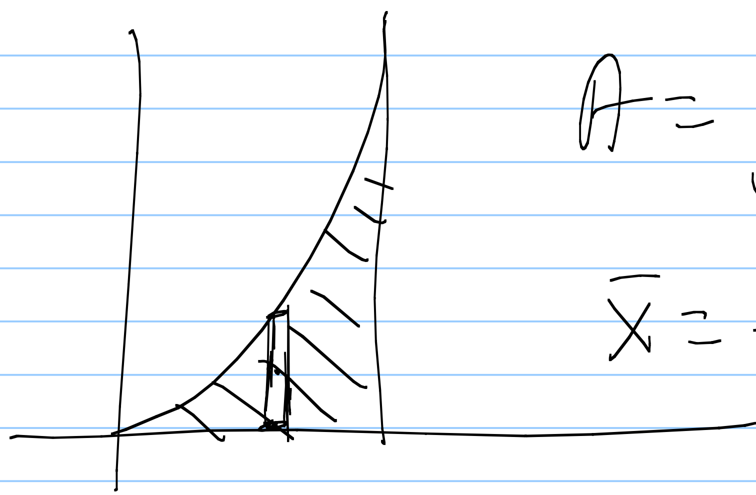
Exam #3

1) $(0,0)$, $(180,210)$, $(-150,60)$ $\quad M = 15\text{ kg}$

$$\bar{x} = \frac{5\cdot 0 + 5\cdot 180 + 5(-150)}{15} = 10\text{ mm}$$

$$\bar{y} = \frac{5\cdot 0 + 5\cdot 210 + 5\cdot 60}{15} = 90\text{ mm}$$

2)



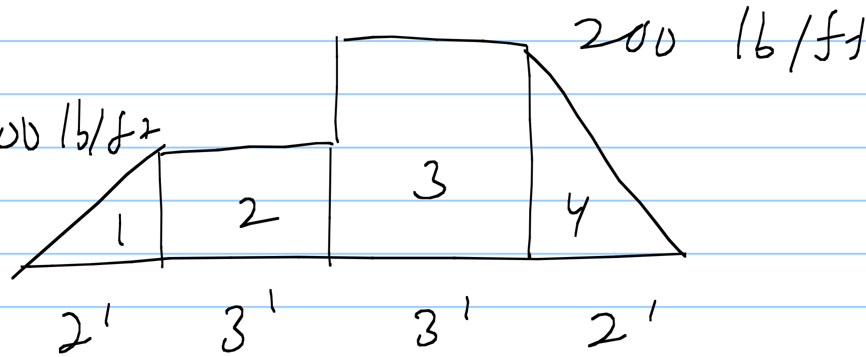
$$A = \int_0^3 x^3\,dx = \frac{x^4}{4}\Big|_0^3 = \frac{81}{4}$$

$$\bar{x} = \frac{1}{A}\int_R x\,dA = \frac{1}{A}\int_0^3 x^4\,dx = \frac{4}{81}\,\frac{x^5}{5}\Big|_0^3 = \frac{4\cdot 3}{5} = \frac{12}{5}$$

$$\bar{y} = \frac{1}{A}\int_0^3 \frac{x^3}{2}\cdot x^3\,dx = \frac{2}{81}\int_0^3 x^6\,dx = \frac{2}{81}\cdot\frac{x^7}{7} = \frac{54}{7}$$

$$\bar{y} = \frac{4}{81}\int_0^3\int_0^{x^3} y\,dy\,dx = \frac{4}{81}\int_0^3 \frac{y^2}{2}\Big|_0^{x^3} dx = \frac{2}{81}\int_0^3 x^6\,dx$$

3)



$R_1 = 100 \text{ lbs}$

$R_2 = 300 \text{ lbs}$

$R_3 = 600 \text{ lbs}$

$R_4 = 201 \text{ lbs}$

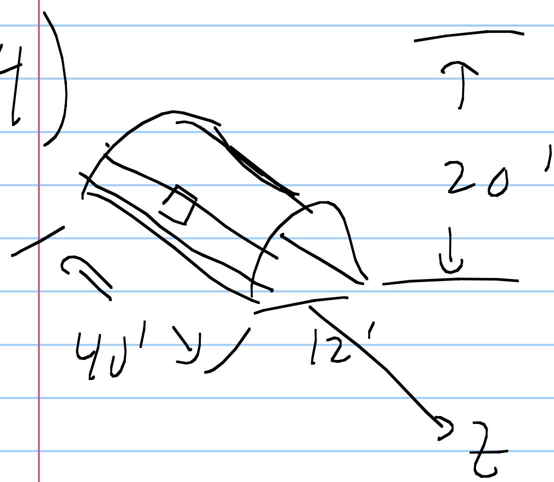
$R = 1200 \text{ lbs}$

$M = 100 \cdot \left(\frac{4}{3}\right) + 300(3.5) + 600(6.5) + 200\left(8\frac{2}{3}\right)$

$= 6812 \text{ ft} \cdot \text{lbs}$

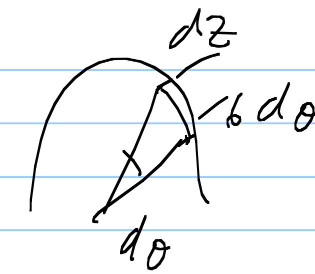
$\bar{x} = \frac{M}{R} = 5.68 \text{ ft}$

4)



$dA = 6\,d\theta\,dz$

$dA = 240\,d\theta$



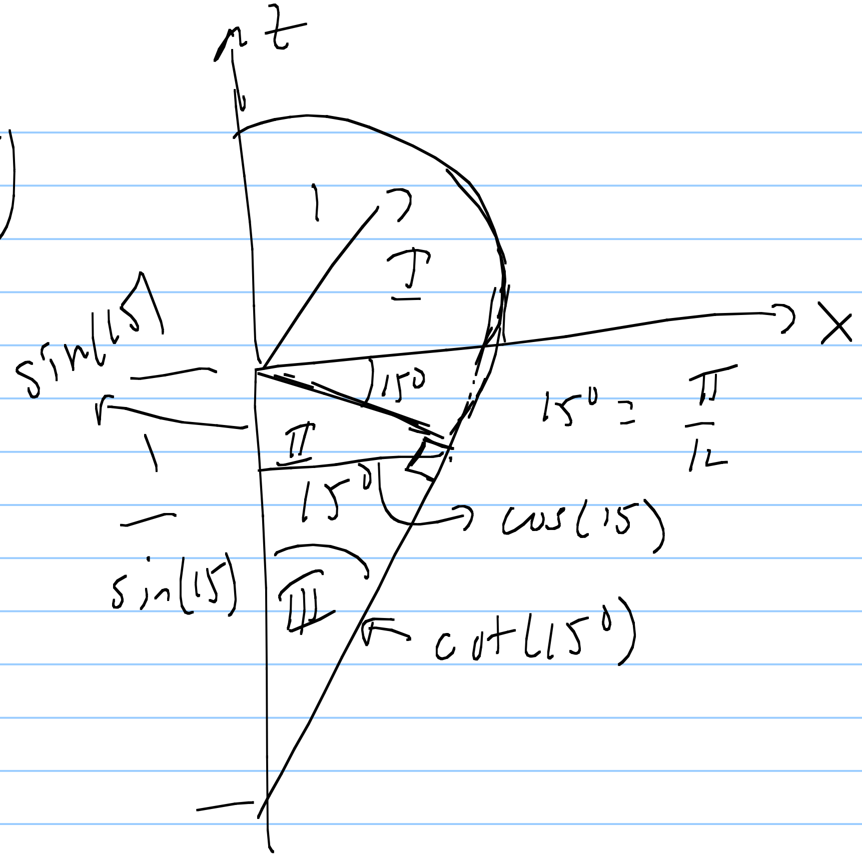
$d = 20 - 6\sin\theta$

$P = \rho d = 62.4(20 - 6\sin\theta)$

$dF = P\,dA \Rightarrow F \int_0^{\pi} 62.4(20 - 6\sin\theta) \cdot 240\,d\theta$

$F = 14976 \int_0^{\pi} (20 - 6\sin\theta)\,d\theta = 14976(20\pi - 12) = 7.61 \times 10^3$ lbs

5)



$$\bar{x} = \bar{y} = 0$$

$$I \Rightarrow V_I = \int_0^{2\pi} \int_0^{\frac{2\pi}{12}} \int_0^1 \rho^2 \, d\rho \sin\phi \, d\phi \, d\theta$$

$$= \frac{2\pi}{3}\left[1 - \cos\frac{2\pi}{12}\right]$$

$$V_{II} = \frac{\pi}{3}\cos^2(15)\sin(15)$$

$$V_{III} = \frac{\pi}{3}\cos^2(15)\left[\csc(15) - \sin(15)\right]$$

$$\bar{z}_I V_I = \int_0^{2\pi}\int_0^{\frac{2\pi}{12}}\int_0^1 \rho^3 \cos\varphi \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \frac{2\pi}{4}\int_0^{\frac{2\pi}{12}} \cos\varphi \sin\varphi \, d\varphi = \frac{\pi}{4} \sin^2\varphi \Big|_0^{2\pi/12} = \frac{\pi}{4}\sin^2\left(\frac{2\pi}{12}\right)$$

$$= \frac{\pi}{4}\cos^2(15)$$

$$\bar{z}_{II} = -\frac{3\sin(15)}{4} \qquad \bar{z}_{III} = -\sin(15) + \frac{1}{4}\left[\csc(15) - \sin(15)\right]$$

$$= -\frac{1}{4}\csc(15) - \frac{3}{4}\sin(15)$$

$$\bar{V} = V_I + V_{II} + V_{III} = \frac{2\pi}{3}\left(1+\sin(15)\right) + \frac{\pi}{3}\cos^2(15)\sin(15)$$

$$+ \frac{\pi}{3}\cos^2(15)\left(\csc(15) - \sin(15)\right)$$

$$= \frac{\pi\left[2\sin(15) + 2\sin^2(15) + \cos^2(15)\right]}{3\sin(15)} = \frac{\pi\left[1 + 2\sin(15) + \sin^2(15)\right]}{3\sin(15)}$$

$$= \frac{\pi\left(1+\sin(15)\right)^2}{3\sin(15)}$$

$$Z U = Z_I V_I + Z_{II} V_{II} + Z_{III} V_{III} = \frac{\pi}{4}\cos^2(15) - \frac{3\sin(15)}{4}\cdot\frac{\pi}{3}\cos^2(15)\sin(15)$$

$$- \left(\frac{1}{4}\csc(15) + \frac{3}{4}\sin(15)\right)\frac{\pi}{3}\cos^2(15)\left[\csc(15) - \sin(15)\right]$$

$$= \frac{\pi}{12}\cos^2(15)\left[3 - 3\sin^2(15) - \left(\csc(15) + 3\sin(15)\right)\left(\csc(15) - \sin(15)\right)\right]$$

$$= \frac{\pi}{12}\cos^2(15)\left[\frac{3}{4} - \frac{3\sin^2(15)}{4} - \csc^2(15) - \frac{3}{4} + 1 + \frac{3\sin^2(15)}{4}\right]$$

$$= \frac{\pi}{12}\cos^2(15)\left[1 - \csc^2(15)\right] = \frac{-\pi\cos^4(15)}{12\sin^2(15)}$$

$$\bar{z} = \frac{-\frac{\pi}{12}\frac{\cos^4(15)}{\sin^2(15)}}{\frac{\pi(1+\sin(15))^2}{3\sin(15)}} = -\frac{\cos^4(15)}{\sin(15)(1+\sin(15))^2} = \frac{-(1-\sin(15))^2}{4\sin(15)}$$

$$= .531$$

Free Body Diagram

2D + 3D Problems

Beam, Frame 3D Frame

4 Problems - 5 Problems