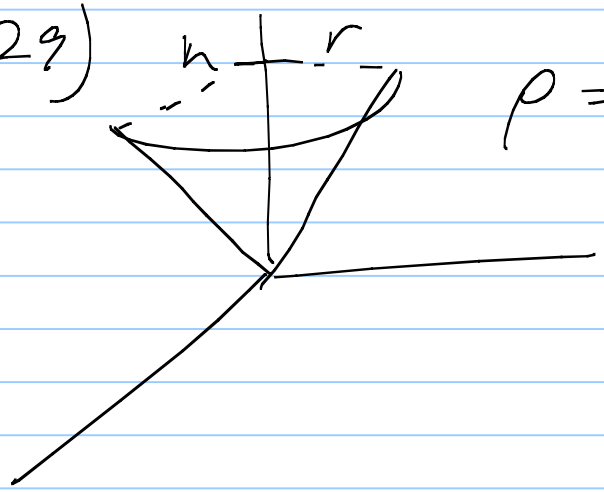
EGR 180 6/22

Note Title 6/22/2009

5.29)



$\rho = \rho_0 z$ $\quad \bar{x} = \bar{y} = 0$

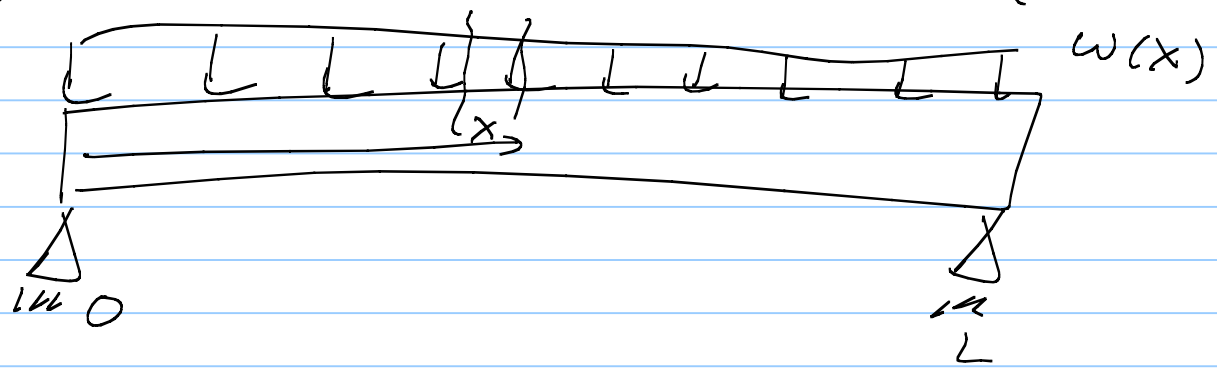
$$M = \iiint_V \rho \, dV = \int_0^{2\pi} \int_0^r \int_0^h \rho_0 z \, dz \, r \, dr \, d\theta$$

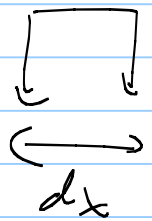
$$= \rho_0 \cdot 2\pi \frac{z^2}{2}\Big|_0^h \frac{r^3}{3}\Big|_0^r = \frac{\rho_0 r^2 h^2 \pi}{2}$$

$$\bar{z} = \frac{1}{m} \iiint_V z \rho \, dV = \frac{2}{\rho_0 \pi h^2 r^2} \int_0^{2\pi} \int_0^r \int_0^h \rho_0 z^2 \, dz \, r \, dr \, d\theta$$

$$= \frac{2}{\rho_0 \pi h^2 r^2} \cdot 2\pi \frac{r^2}{2} \frac{\rho_0}{3} h^3 = \frac{2}{3} h$$

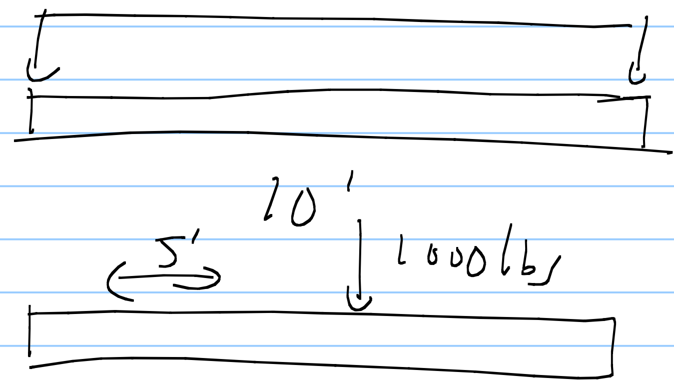
Distributed Loads on a Beam





$$dF = w(x)\,dx \qquad dM = x\,dF$$

$$F = \int_0^L w(x)\,dx \qquad M = \int_0^L x\,w(x)\,dx$$

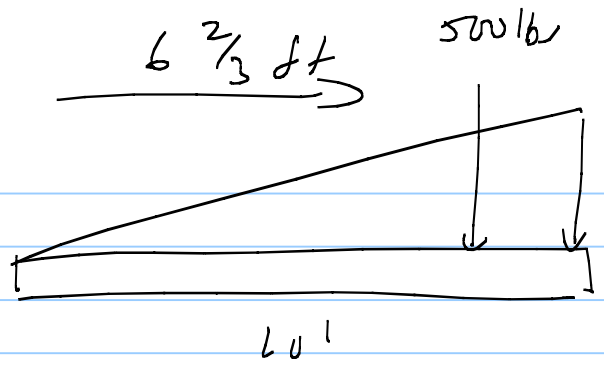


$$w(x) = 100 \text{ lbs/ft}$$

$$F = \int_0^{10} 100\, dx = 1000 \text{ lbs}$$

$$M = \int_0^{10} 100x\, dx = 5000 \text{ ft-lbs}$$

$$\frac{M}{F} = 5 \text{ ft.}$$



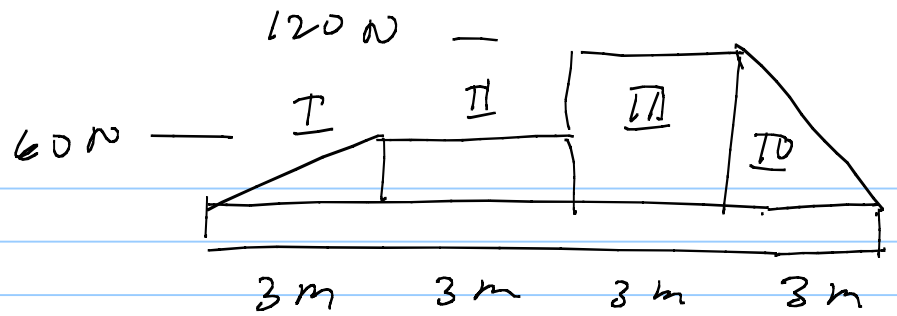
$w(x) = 10x$

$F = \int_0^{10} 10x\,dx = 500 \text{ lbs}$

$M = \int_0^{10} 10x^2\,dx = \frac{10000}{3} \text{ ft-lbs}$

$\bar{x} = \frac{M}{F} \quad \frac{10,000}{3 \cdot 500} = \frac{20}{3}$

$= 6\frac{2}{3}$



$F = 810\ N \qquad \bar{x} = 6\frac{2}{3}\ m$

$M = 5490\ N \cdot m$

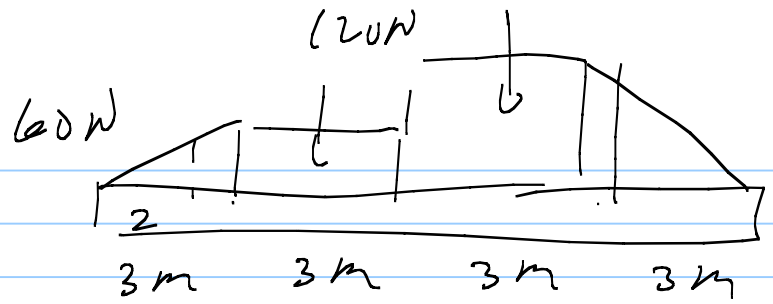
$$F_I = \int_0^3 20x\,dx = 10x^2\Big|_0^3 = 90\ N \qquad M_I = \int_0^3 20x^2\,dx = \frac{20}{3}x^3\Big|_0^3 = 180\ N \cdot m$$

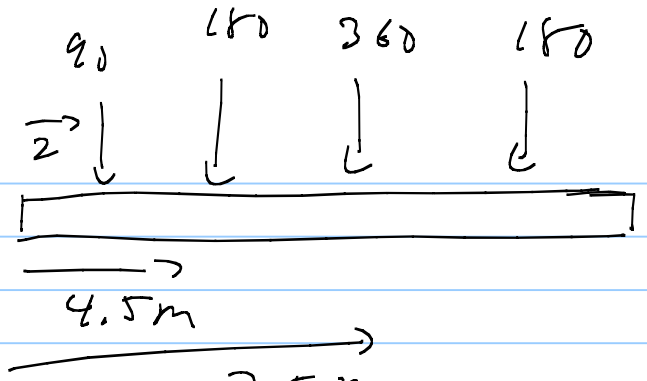
$$F_{II} = \int_3^6 60\,dx = 180\ N \qquad M_{II} = \int_3^6 60x\,dx = 30x^2\Big|_3^6 = 810\ N \cdot m$$

$$F_{III} = \int_6^9 120\,dx = 360\ N \qquad M_{III} = \int_6^9 120x\,dx = 60x^2\Big|_6^9 = 2700\ N \cdot m$$

$$F_{IV} = \int_9^{12} 40(12-x)\,dx = 20(12-x)^2\Big|_{12}^{9} = 180\ N \qquad M_{IV} \int_9^{12} 40(12x-x^2)\,dx$$

$$= 40\left(6x^2 - x^3/3\right)\Big|_9^{12}$$





$F_I = 90\,N$ $\quad m_I = 180\,N\cdot m$

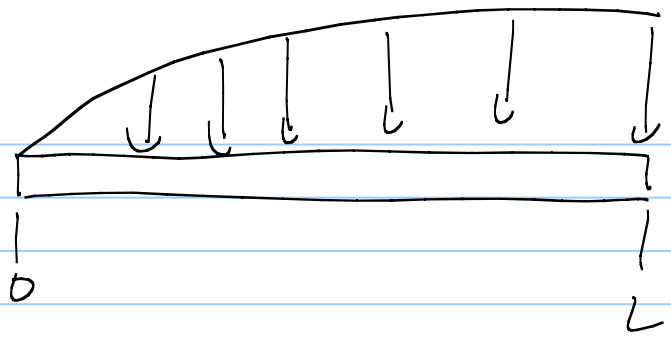
$F_{II} = 180\,N$ $\quad m_{II} = 4.5 \cdot 180 = 810\,N\cdot m$

$F_{III} = 360\,N$ $\quad m_{III} = 2.5 \cdot 360 = 2200\,N\cdot m$

$F_{IV} = 180\,N$ $\quad m_{IV} = 10 \cdot 180 = 1800\,N\cdot m$

$F = 810\,N$ $\quad 5490\,N\cdot m$

$$\bar{x} = \frac{m}{F} = 6\frac{2}{3}\,m$$



$$w(x) = w_0\left[1 - \frac{(L-x)^2}{L^2}\right] = \frac{w_0}{L^2}\left[L^2 - L^2 + 2Lx - x^2\right]$$

$$0 \le x \le L \qquad = \frac{w_0}{L^2}\left[2Lx - x^2\right]$$

F

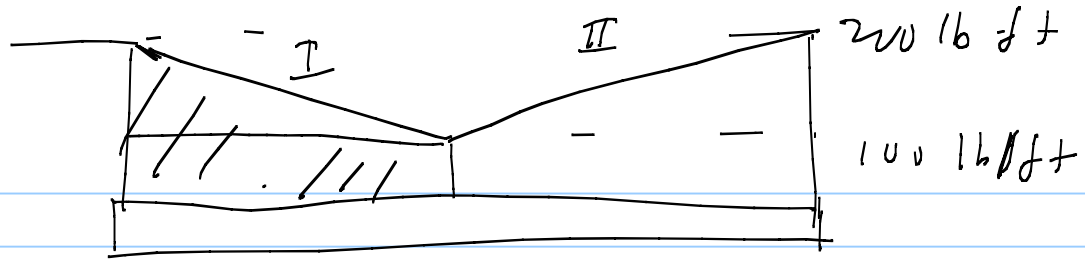
M

$\bar{x}$

$$F = \int_0^L \frac{w_0}{L^2}\left[2Lx - x^2\right) dx = \frac{w_0}{L^2}\left[Lx^2 - \frac{x^3}{3}\right]_0^L = \frac{2w_0 L}{3}$$

$$M = \int_0^L \frac{w_0}{L^2}\left[2Lx - x^2\right) x\, dx = \frac{w_0}{L^2}\left[\frac{2}{3}Lx^3 - \frac{x^4}{4}\right]_0^L = \frac{5w_0 L^2}{12}$$

$$\bar{x} = \frac{M}{F} = \frac{\frac{5w_0 L^2}{12}}{\frac{2w_0 L}{3}} = \frac{5}{12}\cdot\frac{3}{2}L = \frac{5}{8}L$$



F
m
x

$$F_I = \frac{200+100}{2}\cdot 4 = 600 \text{ lbs}$$

$$F_{II} = \frac{100+200}{2}\cdot 6 = 900 \text{ lbs}$$

$$w_I = -25x + 200 \qquad w_{II} = \frac{50}{3}(x-10) + 200 = \frac{50}{3}x - \frac{500}{3} + 200$$

$$M_I = \int_0^4 (-25x^2 + 200x)\,dx \qquad M_{II} = \int_4^{10} \left(\frac{50}{3}x^2 + \frac{100}{3}x\right) dx$$

$$= \frac{-25x^3}{3} + 100x^2 \Big|_0^4 = 1600 - \frac{1600}{3} = \frac{3200}{3} \text{ ft.lbs} \qquad = \frac{50}{9}x^3 + \frac{50}{3}x^2 \Big|_4^{10}$$

$$= \frac{50000}{9} + \frac{5000}{3}$$

$$-\frac{320}{9}-\frac{800}{3}$$

$$=6920 \text{ ft-lbs}$$

$F = 1500$ lbs

$m = 7986\frac{2}{3}$ ft-lbs

$\bar{x} = 5.32$ ft