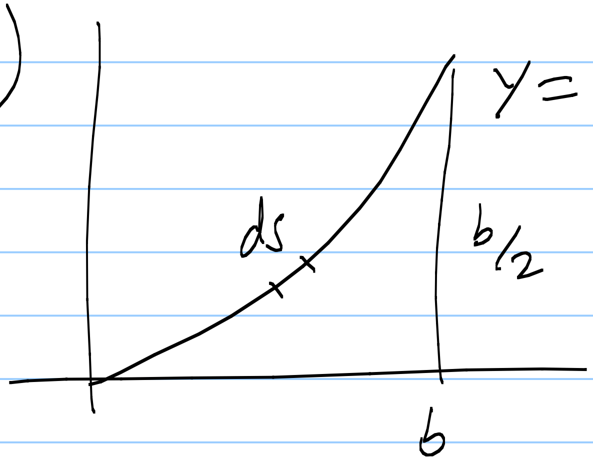
EGR180 6/18

Note Title 6/18/2009

#5.26)



$y = \frac{x^2}{2b}$

$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{1 + (y')^2}\, dx$$

$y' = \frac{x}{b}$ $\quad ds = \sqrt{1 + \frac{x^2}{b^2}}\, dx$

$$L = \int_0^b \sqrt{1 + \frac{x^2}{b^2}}\, dx = \frac{\frac{1}{2} x \left[b^2 + x^2\right]^{1/2} + \frac{1}{2} b^2 \ln\left[x + \sqrt{b^2 + x^2}\right]}{b} \Bigg|_0^b$$

$$= \frac{\frac{1}{2} \cancel{b} \sqrt{2b^2} + \frac{1}{2} b^{\cancel{2}} \ln\left(\cancel{b}(1 + \sqrt{2})\right) - \frac{1}{2} b^{\cancel{2}} \cancel{\ln(b)}}{\cancel{b}} = \frac{\sqrt{2}}{2} b + \frac{b}{2} \ln($$

$$L = \frac{\sqrt{2}}{2} b + \frac{b}{2} \ln(1+\sqrt{2})$$

$$u = 1 + \frac{x^2}{b^2}$$

$$\bar{X} = \frac{1}{L} \int_C x \, ds = \frac{1}{L} \int_0^b x \sqrt{1 + \frac{x^2}{b^2}} \, dx \qquad du = \frac{2x}{b^2} dx$$

$$x \, dx = \frac{b^2}{2} du$$

$$= \frac{1}{L} \cdot \frac{b^2}{2} \int_1^2 u^{1/2} \, du = \frac{1}{L} \frac{b^2}{2} \frac{2}{3} u^{3/2} \Big|_1^2$$

$$= \frac{1}{L} \frac{b^2}{3} \left[2\sqrt{2} - 1 \right] = \frac{b(2\sqrt{2}-1)}{\frac{\sqrt{2}}{2} + \frac{1}{2} \ln(1+\sqrt{2})}$$

$$\bar{y} = \frac{1}{L}\int_C y\,ds = \frac{1}{L}\int_0^b \frac{x^2}{2b}\sqrt{1+\frac{x^2}{b^2}}\,dx$$

$x = b\sinh(t)$

$dx = b\cosh(t)\,dt$

$$= \frac{1}{L}\,\frac{1}{2b}\int_0^{\sinh^{-1}(1)} b^2\sinh^2(t)\cosh(t)\,b\cosh t\,dt$$

$$= \frac{b^2}{2L}\int_0^{\sinh^{-1}(1)} \sinh^2(t)\cosh^2(t)\,dt$$

$\sinh(t)\cosh(t)$

$= \frac{1}{2}\sinh(2t)$

$$= \frac{b^2}{2L}\int_0^{\sinh^{-1}(1)} \frac{1}{4}\sinh^2(2t)\,dt$$

$\sinh^2(2t)$

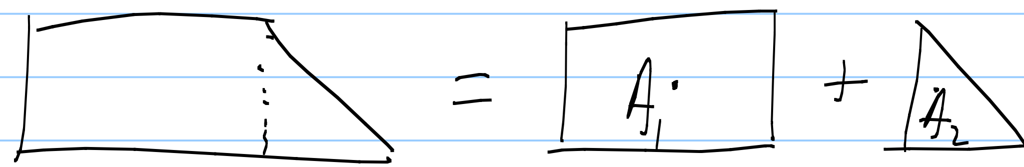
$= \frac{\cosh(4t)-1}{2}$

$$= \frac{b^2}{16L}\int_0^{\sinh^{-1}(1)} (\cosh(4t)-1)\,dt$$

$$\bar{y} = \frac{b^2}{16L}\left[\frac{1}{4}\sinh(4t) - t\,\Big|_0^{\sinh^{-1}(1)}\right]$$

$$= \frac{b^2}{16L}\left[\frac{1}{4}\sinh\left(4\sinh^{-1}(1)\right) - \sinh^{-1}(1)\right]$$

Centroids of Composite Bodies

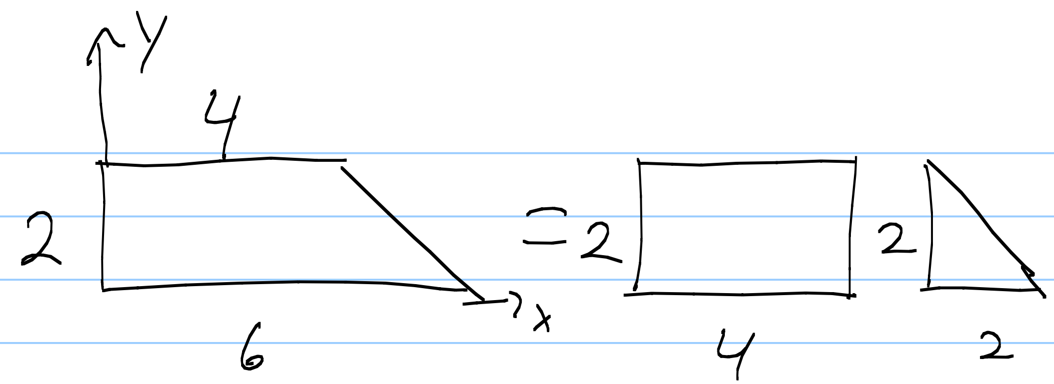


$$\bar{x} = \frac{1}{A}\iint_A x\,dA = \frac{1}{A}\left(\iint_{A_1} x\,dA + \iint_{A_2} x\,dA\right)$$

$$= \frac{1}{A}\left[\bar{x}_1 A_1 + \bar{x}_2 A_2\right] = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A_1 + A_2}$$

$$\bar{y} = \frac{1}{A}\iint_A y\,dA$$

$$A = A_1 + A_2$$

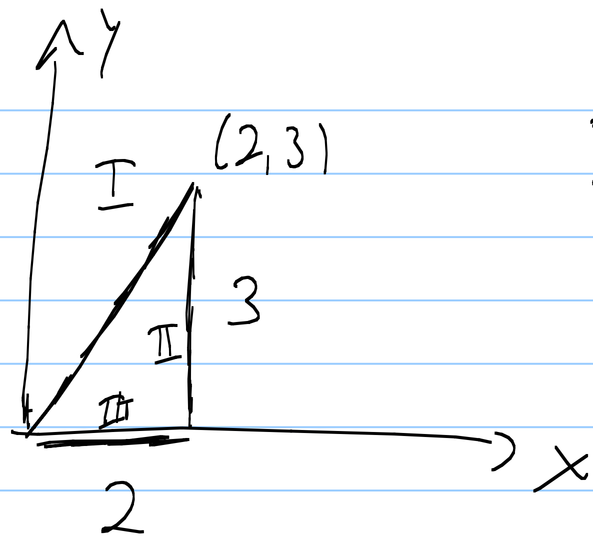


$$\bar{X}_R = 2 \qquad \bar{X}_T = \frac{2}{3} + 4 = \frac{14}{3}$$

$$\bar{X} = \frac{2 \cdot 8 + \frac{14}{3} \cdot 2}{8+2} = \frac{16 + \frac{28}{3}}{10} = \frac{76}{30} = \frac{38}{15}$$

$$\bar{Y}_R = 1 \qquad \bar{Y}_T = \frac{2}{3}$$

$$\bar{Y} = \frac{1 \cdot 8 + \frac{2}{3} \cdot 2}{10} = \frac{8 + \frac{4}{3}}{10} = \frac{28}{30} = \frac{14}{15}$$



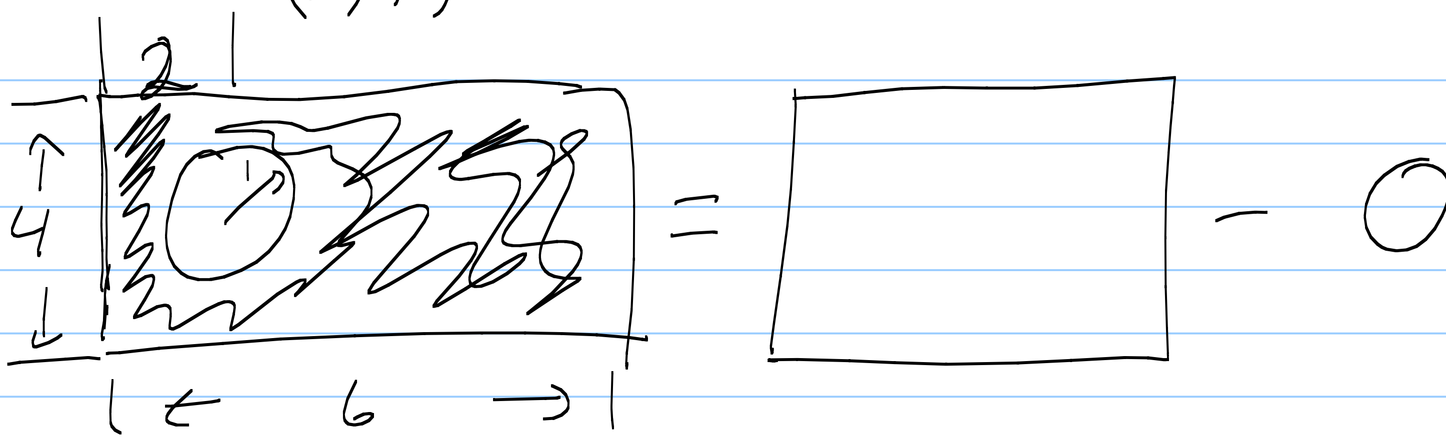
$$\bar{x}_{I} = 1, \quad \bar{x}_{II} = 2, \quad \bar{x}_{III} = 1$$

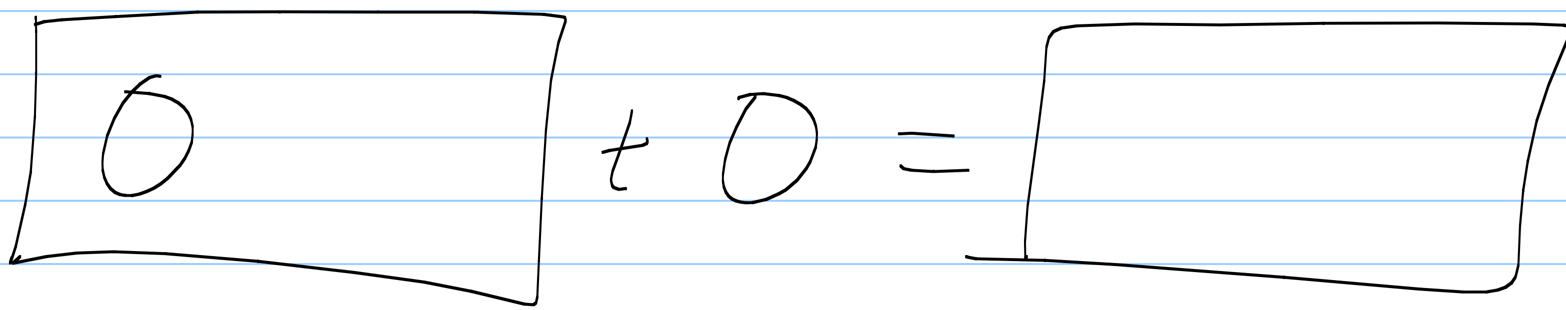
$$\bar{x} = \frac{1\sqrt{13} + 2\cdot 3 + 1\cdot 2}{\sqrt{13} + 3 + 2} = \frac{8+\sqrt{13}}{5+\sqrt{13}}$$

$$\bar{y}_{I} = \frac{3}{2}, \quad \bar{y}_{II} = \frac{3}{2}, \quad \bar{y}_{III} = 0$$

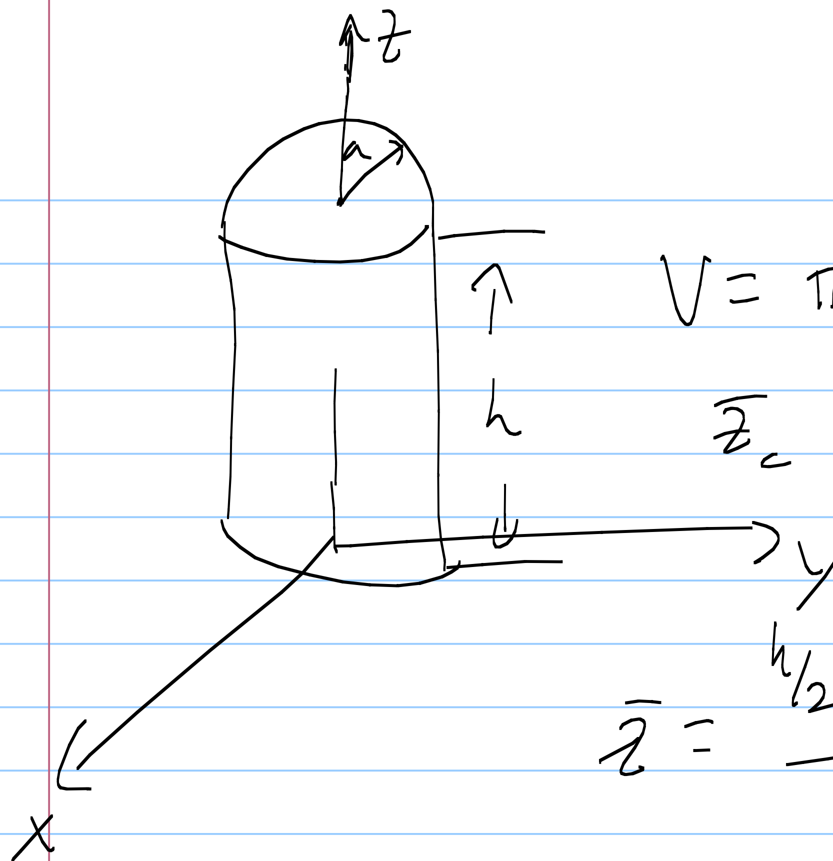
$$\bar{y} = \frac{\frac{3}{2}\sqrt{13} + \frac{3}{2}\cdot 3 + 0\cdot 2}{5+\sqrt{13}} = \frac{9+3\sqrt{13}}{10+2\sqrt{13}}$$

$(\bar{x}, \bar{y})$





$$\bar{x} = \frac{3 \cdot 24 - 2 \cdot \pi}{24 - \pi} = \frac{72 - 2\pi}{24 - \pi} \qquad \bar{y} = \frac{2 \cdot 24 - 2 \cdot \pi}{24 - \pi} = 2$$



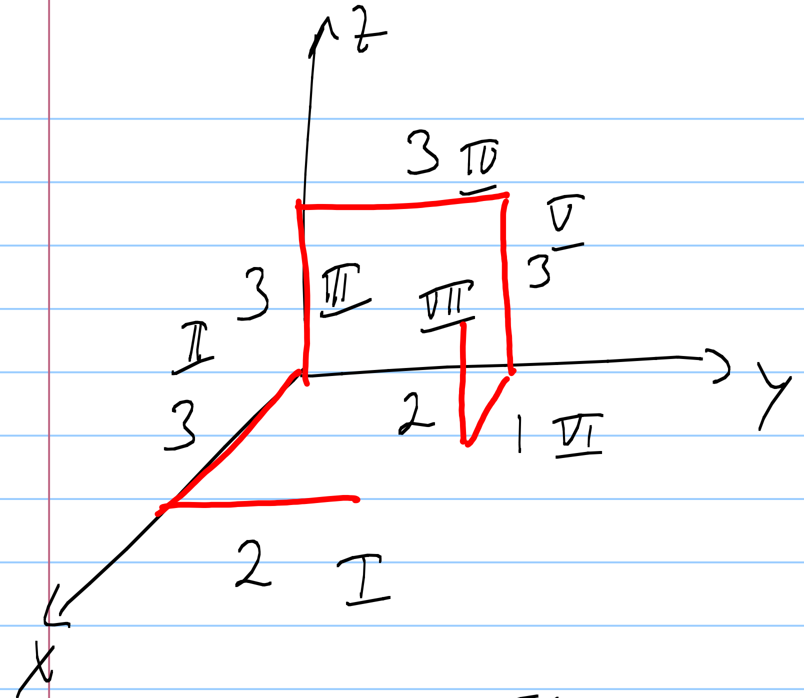
$$V = \pi a^2 h + \frac{2\pi}{3} a^3 \qquad \bar{x} = \bar{y} = 0$$

$$\bar{z}_c = \frac{h}{2} \qquad \bar{z}_s = \frac{3a}{8} + h$$

$$\bar{z} = \frac{\frac{h}{2} \cdot \cancel{\pi a^2} h + \left(\frac{3a}{8} + h\right) \frac{2\cancel{\pi}}{3} a^{\cancel{3}}}{\cancel{\pi a^2} h + \frac{2\cancel{\pi}}{3} a^{\cancel{3}}} \frac{24}{24}$$

$$= \frac{12h^2 + 2(3a + 8h)a}{24h + 16a} = \frac{12h^2 + 16ah + 6a^2}{24h + 16a}$$

$$= \frac{6h^2 + 8ah + 3a^2}{12h + 8a}$$



$\bar{x}_1 = 3 \quad \bar{y}_1 = 1, \quad \bar{z}_1 = 0$

$\bar{x}_2 = 1.5, \quad \bar{y}_2 = 0, \quad \bar{z}_2 = 0$

$\bar{x}_3 = \bar{y}_3 = 0, \quad \bar{z}_3 = 1.5$

$\bar{x}_4 = 0, \quad \bar{y}_4 = 1.5, \quad \bar{z}_4 = 3$

$\bar{x}_5 = 0, \quad \bar{y}_5 = 3, \quad \bar{z}_5 = 1.5$

$\bar{x}_6 = .5, \quad \bar{y}_6 = 3, \quad \bar{z}_6 = 0$

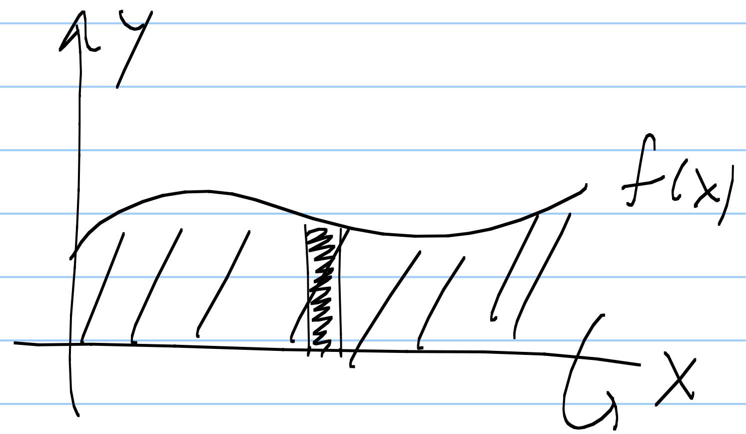
$\bar{x}_7 = 1, \quad \bar{y}_7 = 3, \quad \bar{z}_7 = 1$

$$\bar{X} = \frac{3\cdot 2 + 1.5\cdot 3 + .5\cdot 1 + 1\cdot 2}{17} = \frac{6 + 4.5 + .5 + 2}{17} = \frac{13}{17}$$

$$\bar{Y} = \frac{2 + 4.5 + 9 + 3 + 6}{17} = \frac{24.5}{17} = \frac{49}{34}$$

$$\bar{z} = \frac{1.5\cdot 3 + 3\cdot 3 + 1.5\cdot 3 + 1\cdot 2}{17} = \frac{4.5 + 9 + 4.5 + 2}{17} = \frac{20}{17}$$

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{13}{17}, \frac{49}{34}, \frac{20}{17}\right)$$



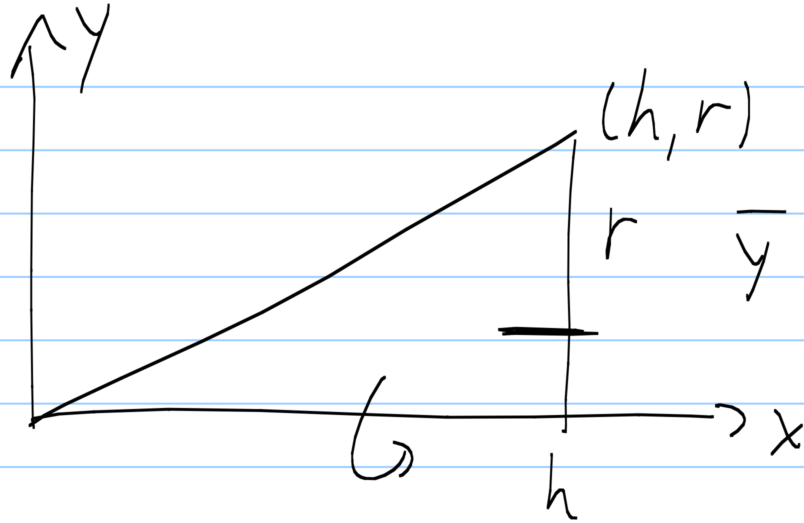
$$dV = \boxed{\pi\left[f(x)\right]^2 dx}$$

$$V = \pi \int_a^b \left[f(x)\right]^2 dx = \pi \int_a^b y\, dA$$

$$= \pi\, \bar{y}\, A$$

$$dA = y\, dx$$

$$\int_a^b y\, dA = \bar{y} \cdot A$$



$$\bar{y} = \frac{r}{3}$$

$$V = \pi \cdot \left(\frac{r}{3}\right)\left[\frac{1}{2} r h\right]$$
$$= \frac{\pi}{6} r^2 h$$