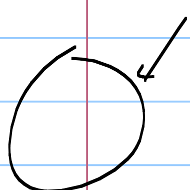
Note Title 6/17/2009

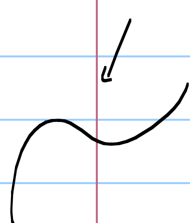
EGR180 6/17

Centroid → Geometric Center of an object.

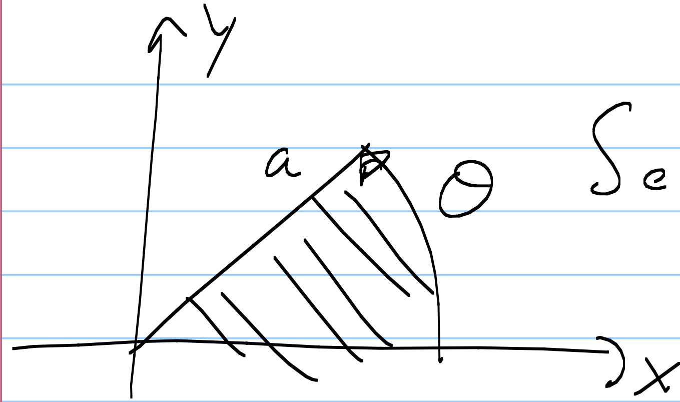
$$\bar{x} = \frac{\iiint_V x\,dV}{V} \qquad \bar{y} = \frac{\iiint_V y\,dV}{V} \qquad \bar{z} = \frac{\iiint_V z\,dV}{V}$$



$$\bar{x} = \frac{\iint_A x\,dA}{A} \qquad \bar{y} = \frac{\iint_A y\,dA}{A} \qquad \bar{z} = \frac{\iint_A z\,dA}{A}$$



$$\bar{x} = \frac{\int_C x\,ds}{L} \qquad \bar{y} = \frac{\int_C y\,ds}{L} \qquad \bar{z} = \frac{\int_C z\,ds}{L}$$



Sector $A = \frac{1}{2} a^2 \theta$

$$\bar{x} = \frac{2}{a^2\theta} \int_0^\theta \int_0^a r\cos\theta \, r \, dr \, d\theta$$

$$= \frac{2}{a^2\theta} \int_0^\theta \cos\theta \, d\theta \cdot \int_0^a r^2 \, dr$$

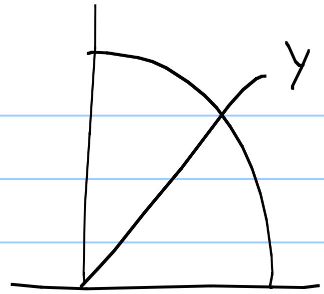
$$= \frac{2}{a^2\theta} \sin\theta \, \frac{a^3}{3} = \frac{2a\sin\theta}{3\theta}$$

$$\bar{y} = \frac{2}{a^2\theta} \int_0^\theta \int_0^a r\sin\theta \, r \, dr \, d\theta$$

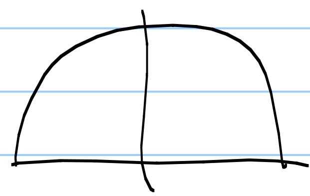
$$= \frac{2}{a^2\theta} \int_0^\theta \sin\theta \, d\theta \int_0^a r^2 \, dr$$

$$= \frac{2}{a^2\theta} \left[1 - \cos\theta\right] \frac{a^3}{3}$$

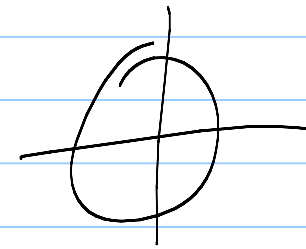
$$= \frac{2a(1-\cos\theta)}{3\theta}$$



$\bar{y} = \bar{x}$ $\quad \bar{y} = \frac{4a}{3\pi}$ $\quad \bar{x} = \frac{4a}{3\pi}$

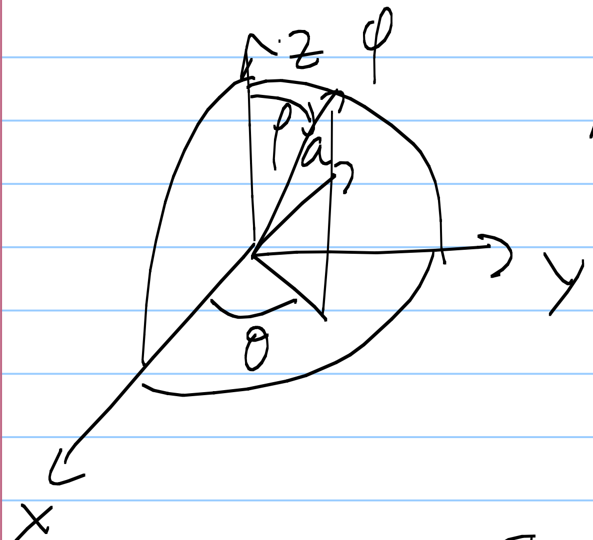


$\bar{x} = 0,\ \bar{y} = \frac{4a}{3\pi},$



$$\int_0^\theta \sin\theta \, d\theta = -\cos\theta \Big|_0^\theta = -\cos\theta - (-1)$$

$$= 1 - \cos\theta$$



Hemisphere $V = \frac{2\pi a^3}{3}$

$x = \rho \sin\varphi \cos\theta$

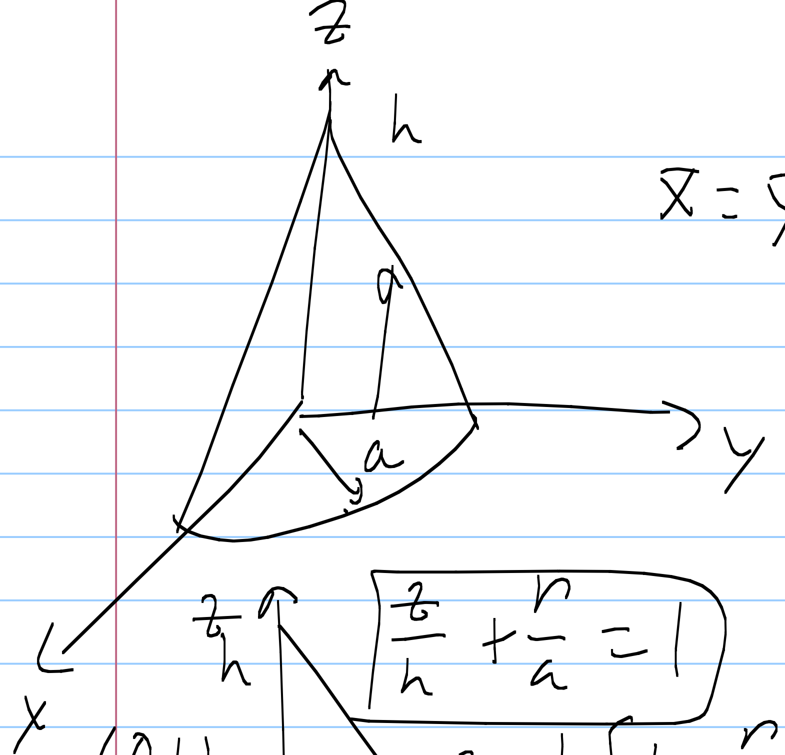
$y = \rho \sin\varphi \sin\theta$ $\qquad dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

$z = \rho \cos\varphi$

$$\bar{z} = \frac{3}{2\pi a^3} \int_0^{2\pi} \int_0^{\pi/2} \int_0^{a} \underline{\rho \cos\varphi \, \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta}$$

$$= \frac{3}{2\pi a^3} \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{2}} \underbrace{\sin\varphi \cos\varphi \, d\varphi}_{\frac{\sin^2\varphi}{2}} \int_0^{a} \underbrace{\rho^3 \, d\rho}_{\frac{\rho^4}{4}} = \frac{3}{\cancel{2\pi} a^{\cancel{3}}} \cancel{2\pi} \cdot \frac{1}{2} \frac{a^{\cancel{4}}}{4}$$

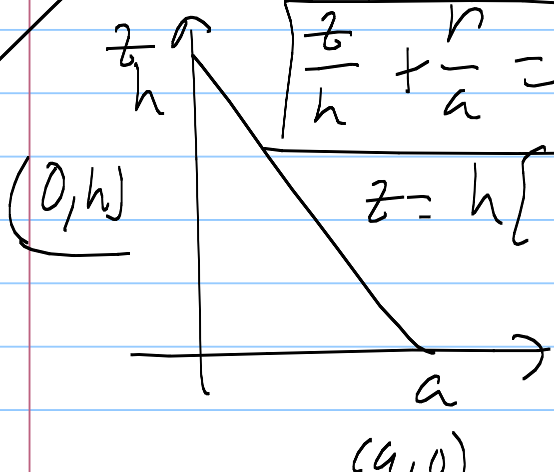
$$= \frac{3a}{8}$$



$\bar{x} = \bar{y} = 0 \qquad V = \frac{\pi}{3} a^2 h$

$$\bar{z} = \frac{3}{\pi a^2 h} \int_0^{2\pi} \int_0^{a} \int_0^{h[1-\frac{r}{a}]} z\, dz\, r\, dr\, \underline{d\theta}$$

$$= \frac{6}{a^2 h} \int_0^a \frac{z^2}{2} \Big|_0^{h(1-\frac{r}{a})} r\, dr$$



$\boxed{\frac{z}{h} + \frac{r}{a} = 1}$

$z = h[1 - \frac{r}{a}]$

$= h[1 - \frac{r}{a}]$

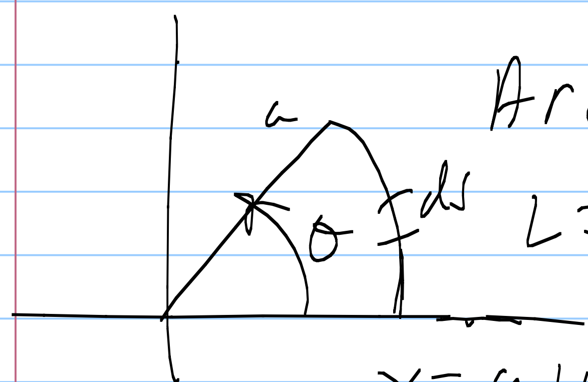
$$= \frac{3}{a^2 h} \int_0^a h^2 (1 - \frac{r}{a})^2 r\, dr$$

$m = \frac{h-0}{0-a} = -\frac{h}{a} \qquad z = -\frac{h}{a} r + h$

$$= \frac{3h}{a^2} \int_0^a \left(r - \frac{2r^2}{a} + \frac{r^3}{a^2}\right) dr$$

$$\bar{z} = \frac{3h}{a^2}\left[\frac{r^2}{2} - \frac{2r^3}{3a} + \frac{r^4}{4a^2}\right]_0^a = \frac{3h}{a^2}\left[\frac{a^2}{2} - \frac{2a^2}{3} + \frac{a^2}{4}\right]$$

$$= \frac{3h}{12}\left[6 - 8 + 3\right] = \frac{h}{4}$$



Arc

$L = a\theta$

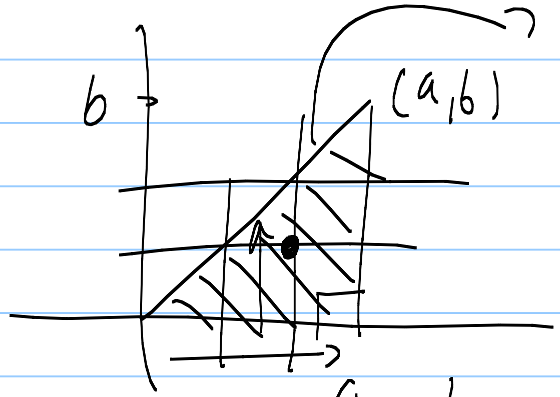
$x = a\cos\theta$

$y = a\sin\theta$

$$\bar{x} = \frac{1}{a\theta}\int_0^\theta a\cos\theta \, a\,d\theta$$

$$= \frac{a}{\theta}\int_0^\theta \cos\theta\,d\theta = \frac{a\sin\theta}{\theta}$$

$$\bar{y} = \frac{1}{a\theta} \cdot \int_0^\theta a\sin\theta \, a\,d\theta = \frac{a}{\theta}\int_0^\theta \sin\theta\, d\theta = \frac{a}{\theta}(1-\cos\theta)$$



$$y = \frac{b}{a}x$$

$$A = \frac{1}{2}ab$$

$$\bar{x} = \frac{2}{ab}\int_0^a \int_0^{\frac{b}{a}x} x\,dy\,dx$$

$$= \frac{2}{ab}\int_0^a \frac{b}{a}x^2\,dx$$

$$= \frac{2}{a^2}\cdot\frac{x^3}{3}\Big|_0^a = \frac{2}{3}a$$

$$\bar{y} = \frac{2}{ab}\int_0^a \int_0^{\frac{b}{a}x} \underset{\frac{y^2}{2}}{y\,dy}\,dx$$

$$= \frac{2}{ab}\int_0^a \frac{b^2x^2}{2a^2}dx = \frac{b}{a^3}\int_0^a x^2\,dx = \frac{b}{a^3}\,\frac{x^3}{3}\Big|_0^a = \frac{b}{3}$$

Centroid → Geometric Center

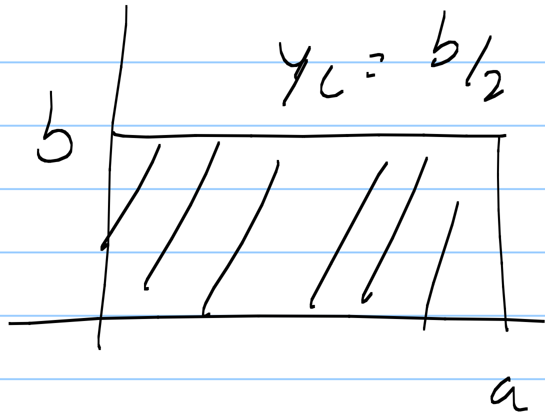
Center of Mass

Density is constant

Center of Mass = Centroid

$x_c = a/2$

$y_c = b/2$



$\rho = \dfrac{xy}{ab}\rho_0$

$M = \iint_A \rho \, dA$

$= \int_0^a \int_0^b \dfrac{\rho_0}{ab} xy \, dy \, dx$

$= \dfrac{\rho_0}{ab} \int_0^a x \, dx \cdot \int_0^b y \, dy$

$= \dfrac{\rho_0}{ab} \dfrac{a^2}{2} \cdot \dfrac{b^2}{2} = \dfrac{\rho_0 ab}{4}$

$\bar{X} = \dfrac{4}{\cancel{\rho_0} ab} \int_0^a \int_0^b \dfrac{\cancel{\rho_0}}{ab} xy \, x \, dy \, dx = $

$= \dfrac{4}{a^2 b^2} \int_0^a x^2 dx \int_0^b y \, dy$

$= \dfrac{\cancel{4}^2}{a^2 \cancel{b}} \dfrac{a^3}{3} \cdot \dfrac{\cancel{b^2}}{\cancel{2}} = \dfrac{2a}{3}$

$\bar{Y} = \dfrac{4}{\cancel{\rho_0} ab} \int_0^a \int_0^b \dfrac{\cancel{\rho_0}}{ab} xy \, y \, dy \, dx$

$\bar{Y} = \dfrac{4}{\cancel{a^2} \cancel{b^2}} \dfrac{\cancel{a^2}}{2} \dfrac{b^3}{3} = \dfrac{2b}{3}$

$$\bar{x} = \frac{1}{m} \iint_A \rho \, x \, dA \qquad \bar{y} = \frac{1}{m} \iint_A \rho \, y \, dA$$