

EGR180

6/16/10

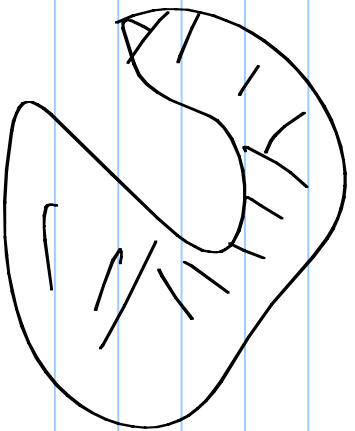
Note Title

6/16/2010

Distributed Forces, Centroids and

Center of Mass

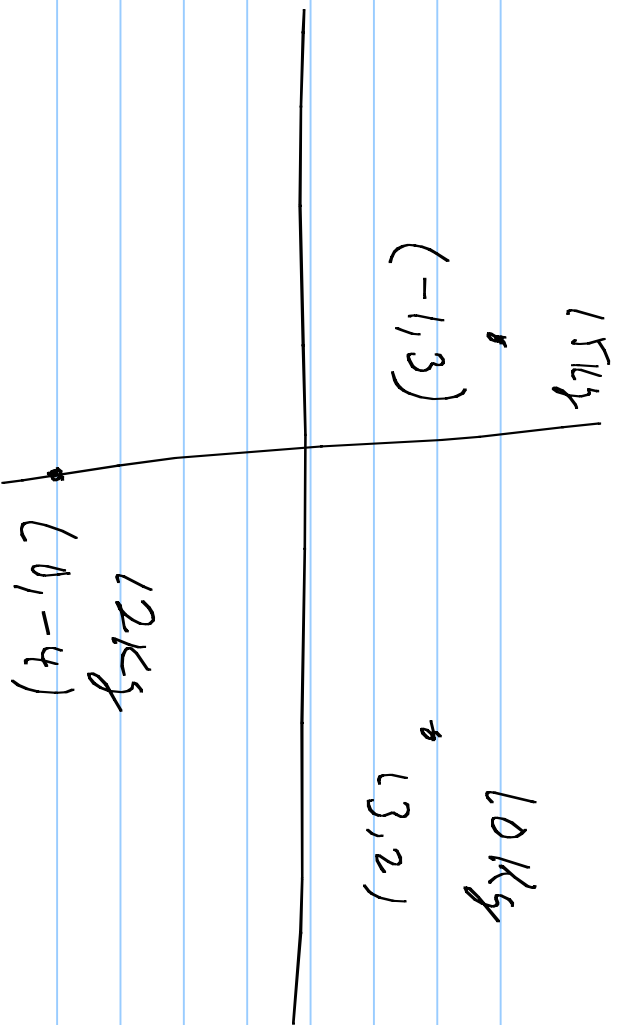
Centroid \equiv Geometric center
of an object



Center of mass \equiv point at which
you can balance
the object.

Centroid \equiv Center of mass density
if the object is constant

Center of mass for a distribution
of point masses

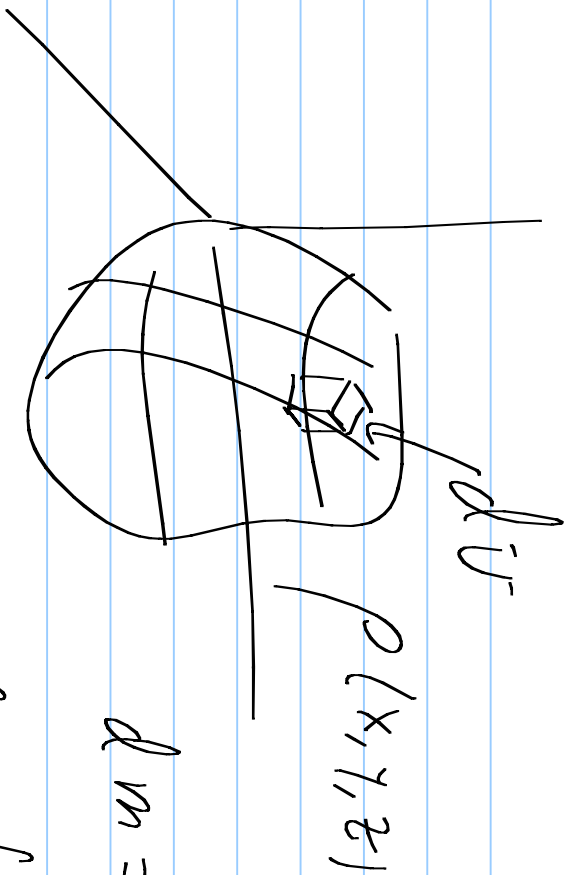


$$\bar{X} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{Y} = \frac{\sum m_i y_i}{\sum m_i}$$

$$\bar{X} = \frac{10(3) + 15(-1) + 12(0)}{10 + 15 + 12} = \frac{30 - 15}{37} = \frac{15}{37}$$

$$\bar{Y} = \frac{10 \cdot 2 + 15 \cdot 3 + 12(-4)}{37} = \frac{20 + 45 - 48}{37} = \frac{17}{37}$$

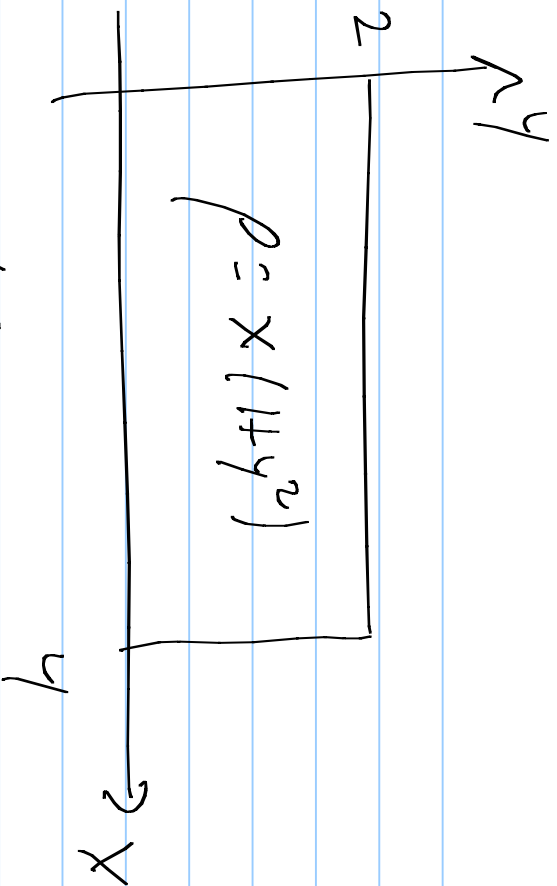


$$dm = \rho dV$$

$$M = \iiint_V \rho dV$$

$$\bar{x} = \frac{\iiint_V x \rho dV}{m}$$

$$\bar{y} = \frac{\iiint_V y \rho dV}{m} \quad \bar{z} = \frac{\iiint_V z \rho dV}{m}$$



$$M = \int_0^4 \int_0^2 x(1+y^2) dy dx = \int_0^4 x dx \cdot \int_0^2 (1+y^2) dy$$

$$= \frac{x^2}{2} \Big|_0^4 \cdot \left[y + \frac{y^3}{3} \right]_0^2 = 8 \cdot \left[2 + \frac{8}{3} \right] = 8 \cdot \frac{14}{3}$$

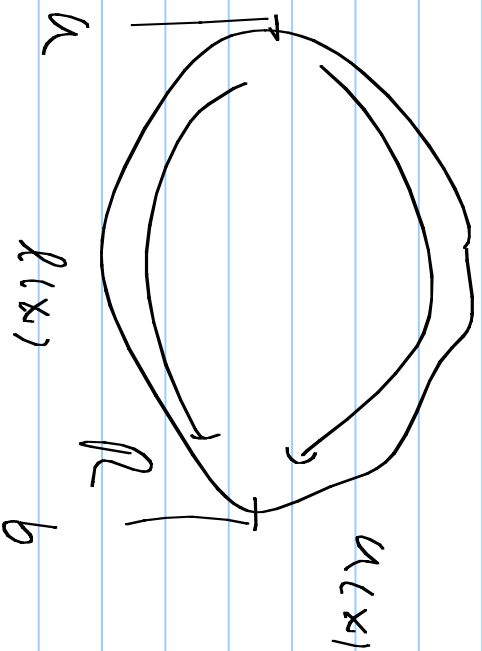
$$= \frac{112}{3}$$

$$\begin{aligned} \bar{X} &= \frac{\iint_R x \, dA}{M} = \frac{3}{112} \int_0^4 \int_0^2 x^2 (1+y^2) \, dy \, dx \\ &= \frac{3}{112} \int_0^4 \left[\frac{x^3}{3} \Big|_0^2 + y + \frac{y^3}{3} \Big|_0^2 \right] dx \\ &= \frac{3}{112} \int_0^4 \left(\frac{8}{3} + y + \frac{8}{3} \right) dx \\ &= \frac{3}{112} \left[\frac{16}{3}x + xy + \frac{8}{3}x \right]_0^4 \\ &= \frac{3}{112} \left(\frac{64}{3} + 4y + \frac{32}{3} \right) \Big|_0^4 \\ &= \frac{3}{112} \left(\frac{96}{3} + 16 \right) = \frac{3}{112} (32 + 16) = \frac{3}{112} \cdot 48 = \frac{3}{7} \end{aligned}$$

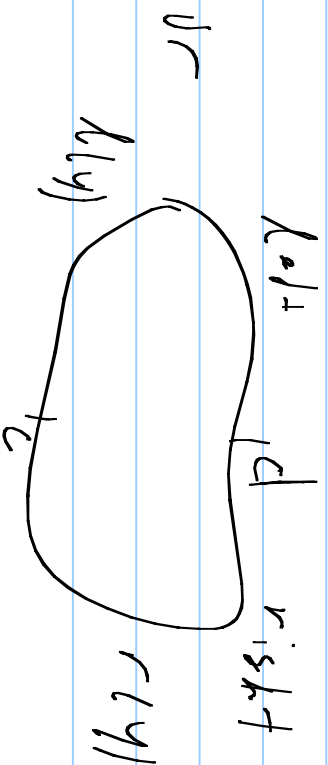
$$\begin{aligned} \bar{y} &= \frac{1}{M} \iint_R y \, dA = \frac{3}{112} \int_0^4 \int_0^2 xy (1+y^2) \, dy \, dx \\ &= \frac{3}{112} \int_0^4 \left[\frac{x^2}{2} \Big|_0^2 + \frac{xy^3}{3} \Big|_0^2 \right] dx \\ &= \frac{3}{112} \int_0^4 \left(\frac{2}{2} + \frac{2y^3}{3} \right) dx \\ &= \frac{3}{112} \left[x + \frac{2xy^3}{3} \right]_0^4 \\ &= \frac{3}{112} \left(4 + \frac{8y^3}{3} \right) \Big|_0^4 \\ &= \frac{3}{112} \left(4 + \frac{128}{3} \right) = \frac{3}{112} \cdot \frac{140}{3} = \frac{140}{112} = \frac{5}{4} \end{aligned}$$

$$= \frac{3}{14} \cdot 8 \cdot (2+4) = \frac{9}{7}$$

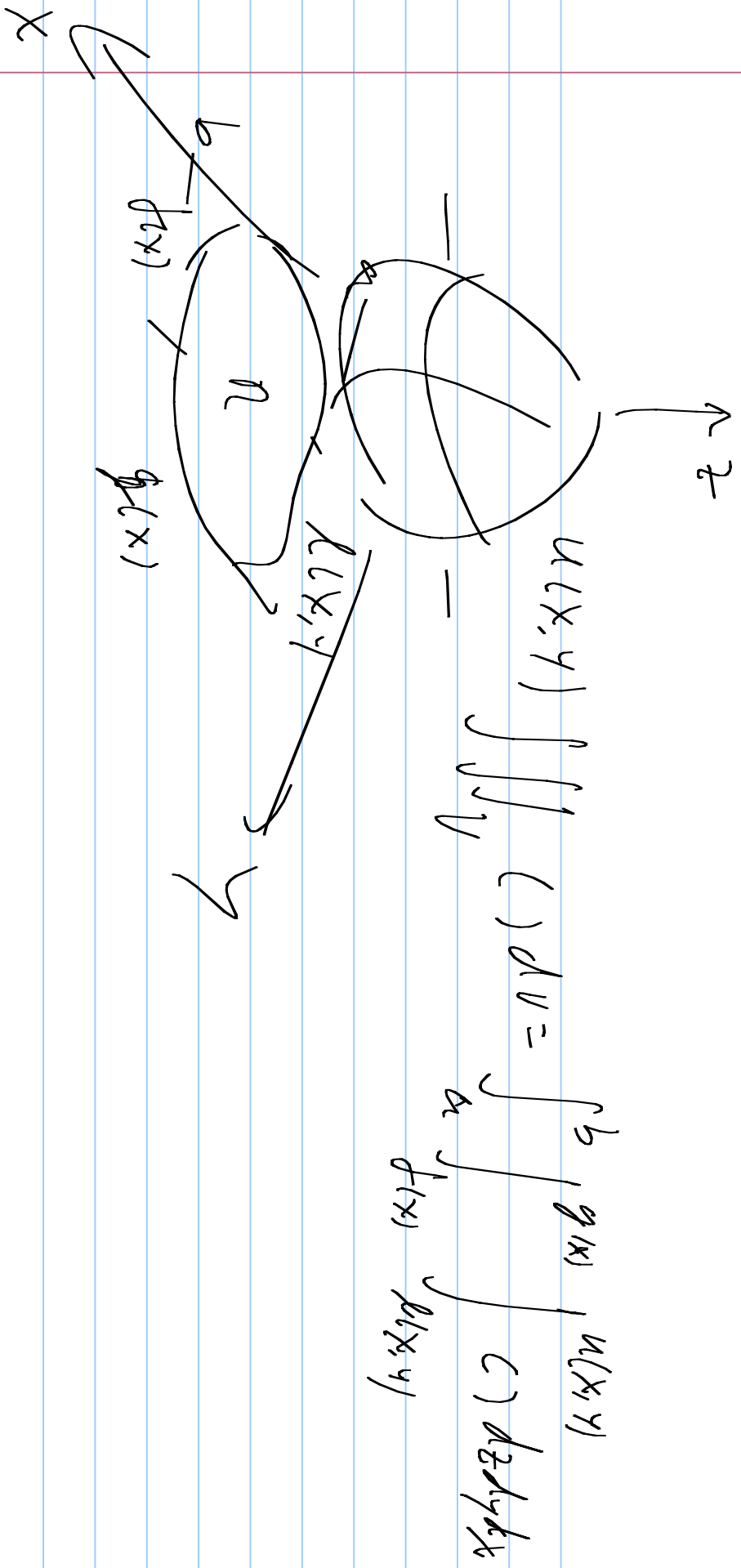
Double Integration



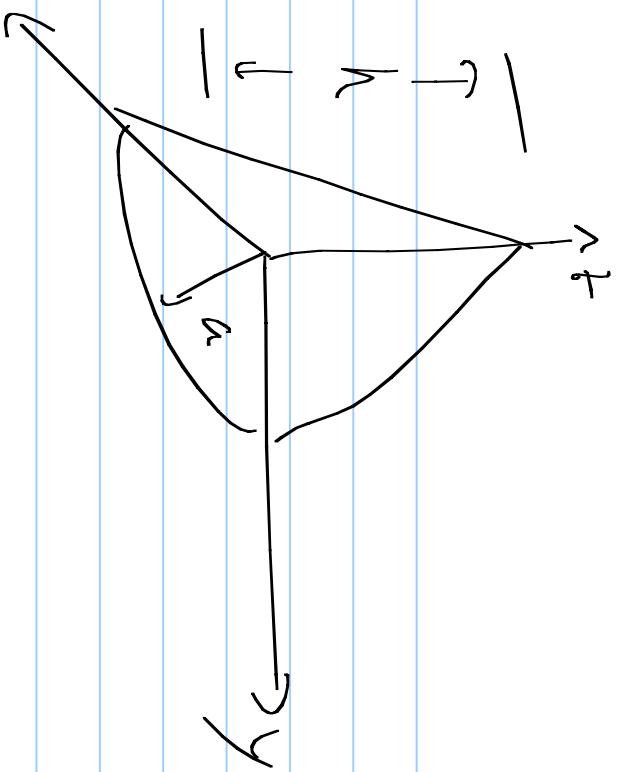
$$\iint_R () dA = \int_a^b \int_{l(x)}^{u(x)} () dy dx$$



$$\iint_R () dA = \int_c^d \int_{l(y)}^{r(y)} () dx dy$$



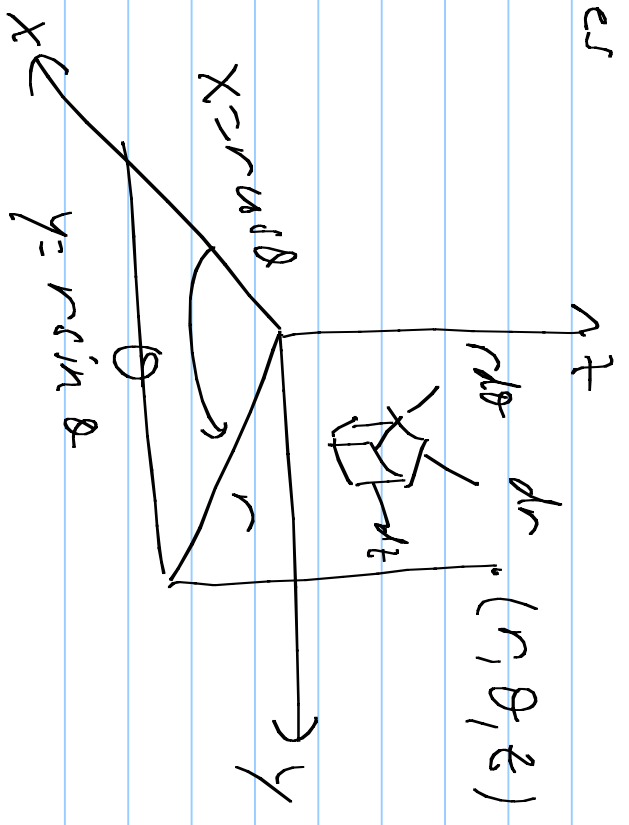
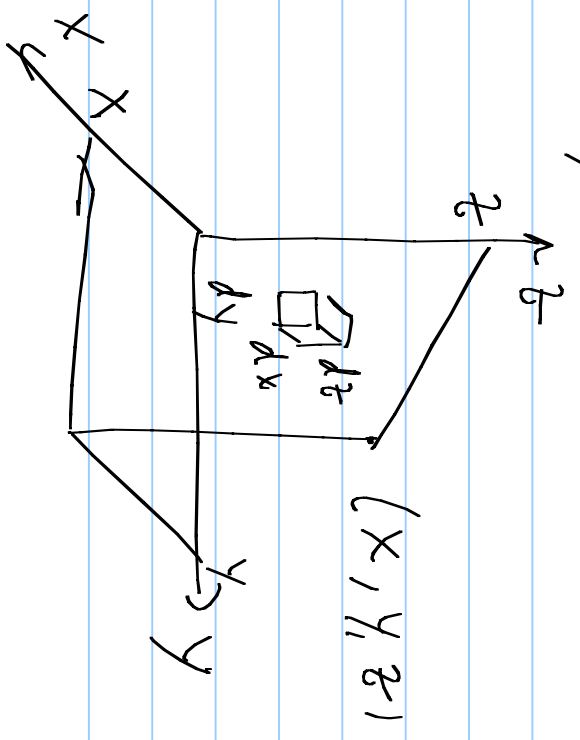
Centroid of a Cone



$$V = \frac{\pi}{3} a^2 h$$

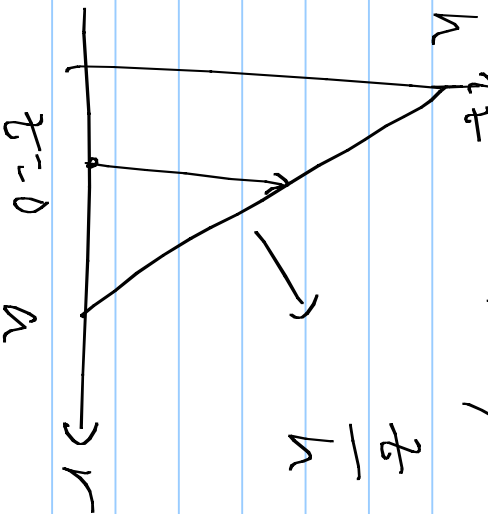
$$x_c = \frac{\iiint_V x \, dV}{V}$$

Cylindrical Coordinates



$$dV = dx dy dz$$

$$dV = r dr d\theta dz$$



$$\frac{z}{h} + \frac{r}{a} = 1 \Rightarrow z = h \left(1 - \frac{r}{a} \right)$$

$$z=0 \quad a \quad \int_0^{2\pi} \int_a^h h \left(1 - \frac{r}{a} \right)$$

$$X \quad dz \quad r dr \quad d\theta$$

$$X_c = \frac{3}{\pi a^2 h} \int_0^{2\pi} \int_a^h \int_0^h X \quad dz \quad r dr \quad d\theta$$

$$= \frac{3}{\pi a^2 h} \int_0^{2\pi} \int_0^a \int_0^{h(1-\frac{r}{a})} r \cos\theta \quad dz \quad r dr \quad d\theta$$

$$= \frac{3}{\pi a^2 h} \int_0^{2\pi} \int_0^a \cos \theta \, da \cdot \int_0^a \int_0^a h \left(1 - \frac{r}{a}\right) r^2 \, dz \, dr = 0$$

$$z_c = \frac{3}{\pi a^2 h} \int_0^{2\pi} \int_0^a \int_0^a h \left(1 - \frac{r}{a}\right) z \, r \, dz \, dr \, d\theta$$

$$= \frac{b}{a^2 h} \int_0^a \int_0^a h \left(1 - \frac{r}{a}\right) z \, dz \, r \, dr$$

$$= \frac{b}{a^2 h} \int_0^a \frac{z^2}{2} \Big|_0^a h \left(1 - \frac{r}{a}\right) r \, dr$$

$$= \frac{3}{a^2 h} \int_0^a h^2 \left(1 - \frac{r^2}{a}\right)^2 r \, dr$$

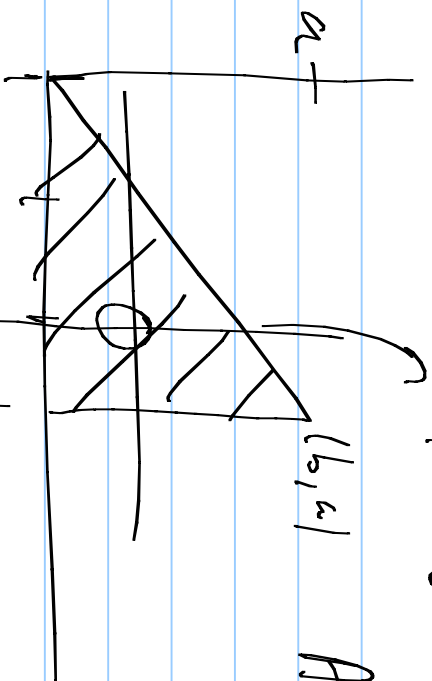
$$= \frac{3h}{a^2} \int_0^a \left(r - \frac{2r^2}{a} + \frac{r^3}{a^2}\right) dr$$

$$= \frac{3h}{a^2} \left[\frac{r^2}{2} - \frac{2r^3}{3a} + \frac{r^4}{4a^2} \right]_0^a$$

$$= \frac{3h}{a^2} \left[\frac{a^2}{2} - \frac{2a^2}{3} + \frac{a^2}{4} \right] = \frac{3h}{12} [6 - 8 + 3]$$

$$= \frac{h}{4}$$

$$y = \frac{a}{b}x$$



$$A = \frac{1}{2}ab$$

$$x_c = \frac{2}{ab} \int_0^b \int_0^{\frac{a}{b}x} x dy dx$$

$$= \frac{2}{ab} \int_0^b \frac{ax^2}{b} dx$$

$$= \frac{2}{b^2} \int_0^b \frac{x^3}{3} \Big|_0^b = \frac{2}{3}b$$

$$y_c = \frac{2}{ab} \int_0^b \int_0^{\frac{a}{b}x} y dy dx$$

$$= \frac{2}{ab} \int_0^b \frac{a^2 x^2}{2b^2} dx$$

$$= \frac{a}{b^3} \int_0^b x^2 dx = \frac{a}{b^3} \frac{x^3}{3} \Big|_0^b = \frac{a}{3}$$