

EGR 180 6/16

Center of Gravity for

Point masses,

(x_3, y_3, z_3)

m_3

(x_1, y_1, z_1)

m_1

(x_2, y_2, z_2)

m_2

X

Z

Y

$$\bar{X} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{Y} = \frac{\sum m_i y_i}{\sum m_i}$$

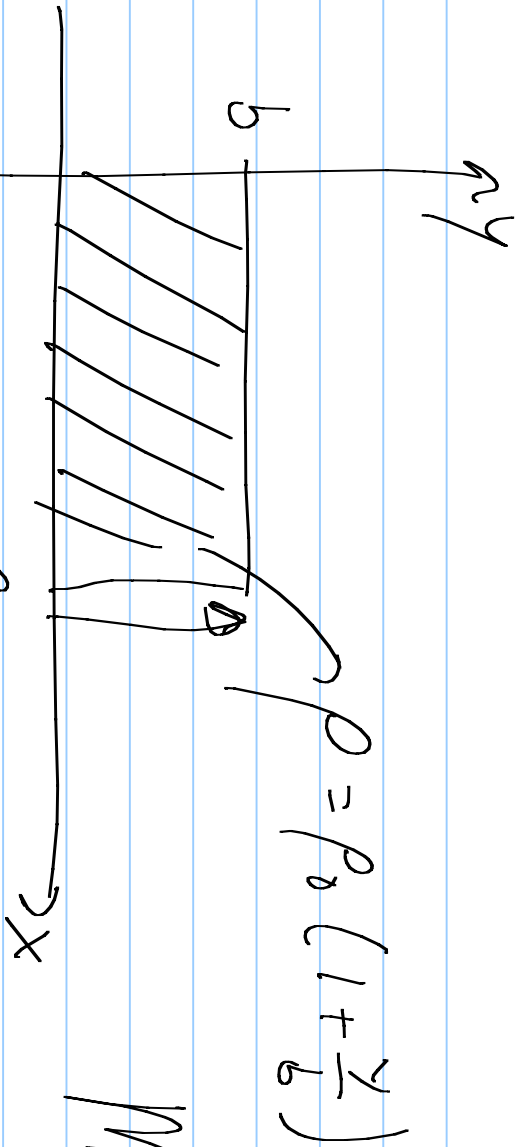
10 kg @ (3, 4) m 20 kg @ (-2, 3) m + 40 kg @ (-1, -6) m

$$\bar{X} = \frac{10 \cdot 3 + 20(-2) + 40(-1)}{10 + 20 + 40} = \frac{-50}{70} = -\frac{5}{7} \text{ m}$$

$$\bar{Y} = \frac{10 \cdot 4 + 20 \cdot 3 + 40(-6)}{70} = \frac{-140}{70} = -2 \text{ m}$$

$\rho(x, y, z)$
 $dm = \rho dV$
 $m = \iiint_V \rho dV$

$$\bar{x} = \frac{\iiint_V x dm}{m} = \frac{\iiint_V \rho x dV}{\iiint_V \rho dV}$$



$$\rho = \rho_0 \left(1 + \frac{y}{b}\right)$$

$$M = \int_0^a \int_0^b \rho_0 \left(1 + \frac{y}{b}\right) dy dx$$

$$\bar{x} = \frac{\int_0^a \int_0^b \rho_0 \left(1 + \frac{y}{b}\right) x dy dx}{\frac{3}{2} \rho_0 a b} = a \rho_0 \left[y + \frac{y^2}{2b} \right]_0^b$$

$$= a \rho_0 \frac{3}{2} b = \frac{3}{2} \rho_0 a b$$

$$= \frac{a^2}{2} \cdot \rho_0 \cdot \frac{3}{2} b$$

$$\frac{3}{2} \rho_0 a b$$

$$= \frac{9}{2}$$

$$y = \frac{\int_0^a \int_0^b \rho_0 \left(1 + \frac{y}{b}\right) y \, dy \, dx}{\frac{3}{2} \rho_0 a b}$$

$$= \frac{\rho_0 \int_0^b \left(y + \frac{y^2}{b}\right) dy}{\frac{3}{2} \rho_0 a b} = \frac{y^2}{2} + \frac{y^3}{3b} \Big|_0^b$$

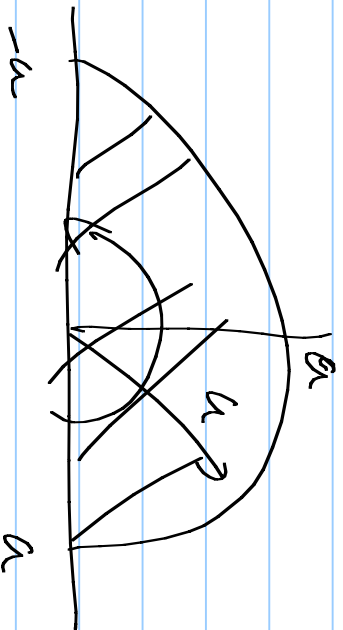
$$= \frac{\frac{3}{2} b}{\frac{3}{2} b}$$

$$= \frac{b^2}{2} + \frac{b^3}{3}$$

$$= \frac{5}{6} b^2$$

$$= \frac{\frac{5}{6} b^2}{\frac{3}{2} b} = \frac{5}{9} b$$

$$= \frac{\frac{3}{2} b}{\frac{3}{2} b}$$



$$\rho = \rho_0 \left(1 + \frac{y}{a} \right)$$

$$M = \int_0^{\pi} \int_0^a \rho_0 \left(1 + \frac{r \sin \theta}{a} \right) r dr d\theta$$

$$M = \frac{\pi}{2} \rho_0 a^2 + \frac{2}{3} \rho_0 a^2 = \rho_0 \int_0^{\pi} \int_0^a \left(r + \frac{r^2}{a} \sin \theta \right) dr d\theta$$

$$= \rho_0 \int_0^{\pi} \left[\frac{r^2}{2} + \frac{r^3}{3a} \sin \theta \right]_0^a d\theta$$

$$= \rho_0 \int_0^{\pi} \left(\frac{a^2}{2} + \frac{a^2}{3} \sin \theta \right) d\theta$$

$$\bar{X} = \frac{\int_0^{\pi} \int_0^a \rho_0 \left(1 + \frac{r \sin \theta}{a} \right) r \cos \theta \, r \, dr \, d\theta}{\left(\frac{\pi}{2} + \frac{2}{3} \right) \rho_0 a^2}$$

$$= \frac{\int_0^{\pi} \int_0^a \left[\frac{r^3}{3} + \frac{r^4}{4a} \sin \theta \right] \cos \theta \, dr \, d\theta}{\left(\frac{\pi}{2} + \frac{2}{3} \right) \rho_0 a^2}$$

$$= a^3 \int_0^{\pi} \left[\frac{\cos \theta}{3} + \frac{\sin \theta \cos \theta}{4} \right] d\theta$$

$$= \frac{a^3 \left[\frac{\pi}{2} + \frac{2}{3} \right]}{\left(\frac{\pi}{2} + \frac{2}{3} \right) \rho_0 a^2}$$

$$= 0$$

$$\bar{Y} = \frac{\int_0^{\pi} \int_0^a R \left(1 + \frac{r \sin \theta}{a} \right) r \sin \theta \, r \, dr \, d\theta}{\int_0^{\pi} \int_0^a R \left(1 + \frac{r \sin \theta}{a} \right) R \, a^2 \, dr \, d\theta}$$

$$= \frac{\int_0^{\pi} \left(\frac{r^3}{3} + \frac{r^4}{4a} \sin \theta \right) \sin \theta \, d\theta \Big|_0^a}{\int_0^{\pi} \left(\frac{r^3}{3} + \frac{r^4}{4a} \sin \theta \right) \sin \theta \, d\theta \Big|_0^a}$$

$$\frac{a^2 \left(\frac{\pi}{2} + \frac{2}{3} \right)}{1 - \cos(2\theta)} \quad \rightarrow \quad \frac{1 - \cos(2\theta)}{2}$$

$$= \frac{a \int_0^{\pi} \left(\frac{1}{3} \sin \theta + \frac{1}{4} \sin^2 \theta \right) d\theta}{\frac{1 - \cos(2\theta)}{2}}$$

$$\frac{\frac{\pi}{2} + \frac{2}{3}}{2}$$

$$= a \left[\frac{2}{3} + \frac{\pi}{8} \right] \cdot 24 = \frac{a(16 + 3\pi)}{16 + 12\pi} = \bar{y}$$

$$\dot{=} 473 a$$

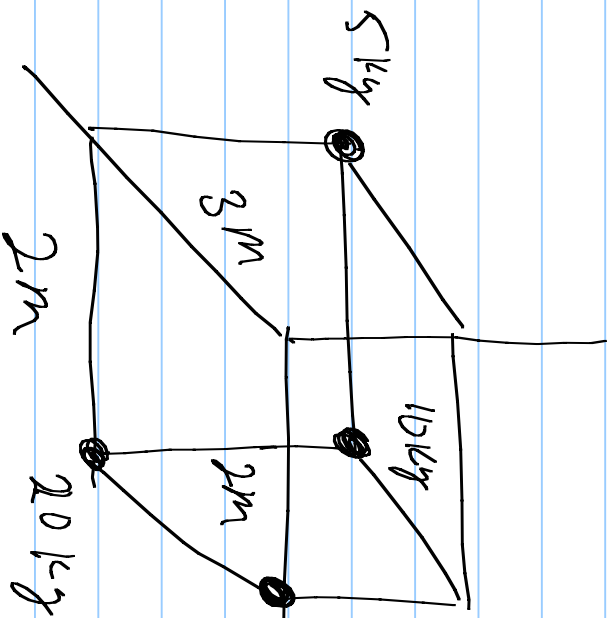
$$\rho = \rho_0$$

$$M = \int_0^\pi \int_0^a \rho_0 r dr d\theta = \frac{\pi \rho_0 a^2}{2}$$

$$\bar{y} = \frac{\int_0^\pi \int_0^a \rho_0 r \sin \theta r dr d\theta \int_0^\pi \sin \theta d\theta \int_0^a r^2 dr}{\frac{\pi \rho_0 a^2}{2} \int_0^\pi \sin \theta d\theta \int_0^a r^2 dr} = \frac{\pi \rho_0 a^2}{2}$$

$$= \frac{2 \cdot \frac{a^3}{3}}{\frac{\pi}{2} a^2} = \frac{4}{3\pi} a = .424 a$$

$$\sum m = 45 \text{ kg}$$



$$\bar{x} = \frac{3.5 + 6.0 - 3 + 2.0 \cdot 3 + 1.0 \cdot 0}{45}$$

$$= \frac{15 + 30 + 6.0}{45} = \frac{51}{45} = \frac{17}{15} = 1.13 \text{ m}$$

$$\bar{y} = \frac{0.5 + 2.1 + 0 + 2.0 + 2.0 + 2.1 \cdot 0}{45}$$

$$= \frac{6.7}{45} = \frac{14}{15} = 0.93 \text{ m}$$

$$\left(\frac{7}{3}, \frac{16}{9}, \frac{2}{3}\right) m$$

$$\begin{aligned} \bar{z} &= \frac{5.2 + 10.2}{45} = \frac{30}{45} \\ &= \frac{2}{3} m \end{aligned}$$