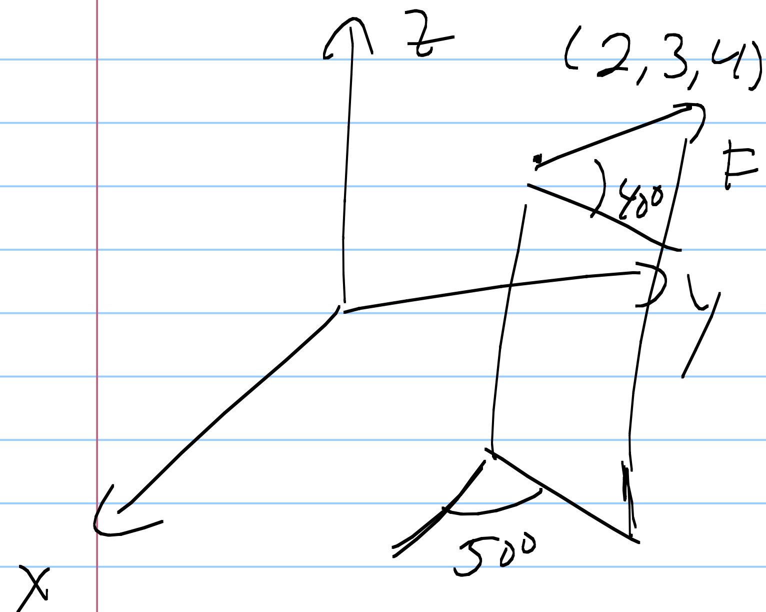
EGR180 6/9

Note Title 6/9/2009

$\vec{M} = \vec{r} \times \vec{F}$ $|\vec{M}| = |\vec{r}||\vec{F}| \sin\theta$



$F = 1500$ lbs

$\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{F} = 1500\cos 40 \cos 50\,\hat{i} + 1500\cos 40 \sin(50)\,\hat{j} + 1500 \sin(40)\,\hat{k}$

$\vec{M} = \vec{r} \times \vec{F}$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1500\cos 40 \cos 50 & 1500\cos 40 \sin 50 & 1500 \sin 40 \end{vmatrix}$$

$$= \hat{i}\left[4500 \sin 40 - 6000 \cos 40 \sin 50\right]$$

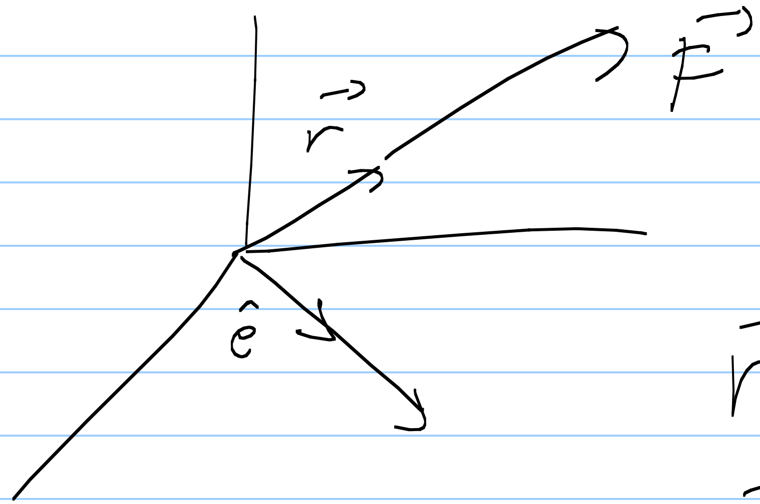
$$- \hat{j}\left[3000 \sin 40 - 6000 \cos 40 \cos 50\right]$$

$$+ \hat{k}\left[3000 \cos 40 \sin 50 - 4500 \cos 40 \cos 50\right]$$

$$= -628.4\,\hat{i} + 1026.1\,\hat{j} - 455.3\,\hat{k}$$

$$M = 1286.5 \text{ ft}\cdot\text{lbs}$$

Moment about a line



$$M_e = (\vec{r} \times \vec{F}) \cdot \hat{e}$$

$$\vec{r} = .5\hat{i} + .4\hat{j} - .8\hat{k} \quad (m)$$

$$\vec{F} = 300\hat{i} + 400\hat{j} + 200\hat{k} \quad (N)$$

$x = y$ in xy plane

$$\hat{e} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$x = t, \quad y = t, \quad z = 0$

$\vec{x} = t\hat{i} + t\hat{j}$

$$\hat{e} = \frac{\vec{x}}{|\vec{x}|} = \frac{t\hat{i} + t\hat{j}}{\sqrt{2}\,t} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .5 & .4 & -.8 \\ 300 & 400 & 200 \end{vmatrix}$$

$$= \hat{i}(400) - \hat{j}(340) + \hat{k}(80)$$

$$M_e = \hat{e} \cdot \vec{M}$$

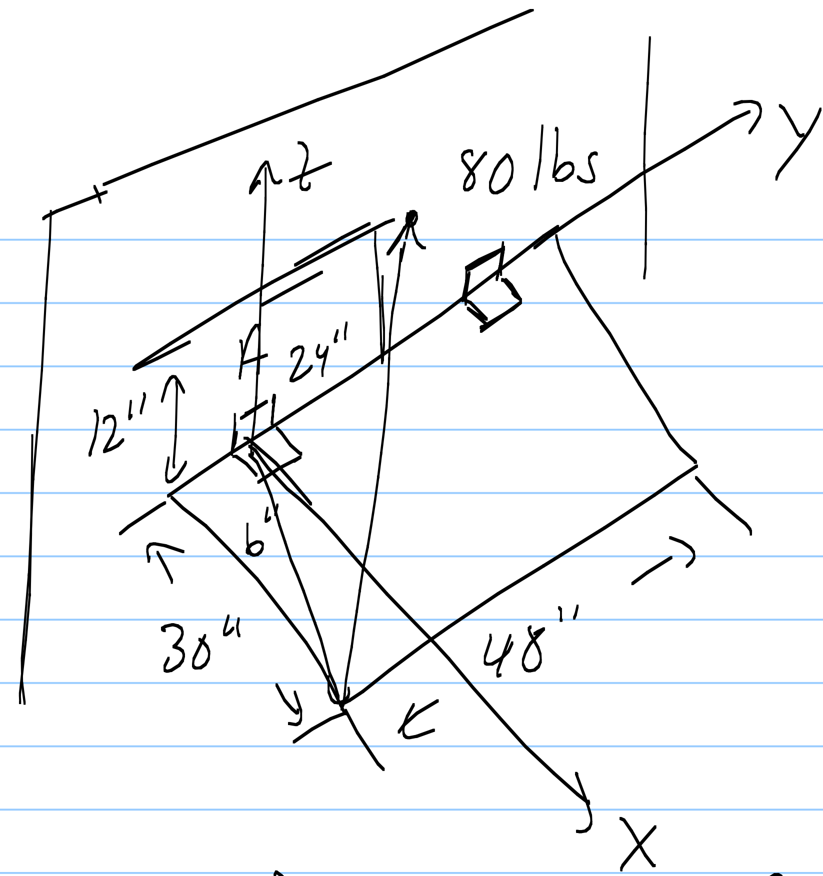
$$M_e = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) \cdot \left(400\hat{i} - 340\hat{j} + 80\hat{k}\right)$$

$$= \frac{60}{\sqrt{2}} = 30\sqrt{2}\ N\cdot m = 42.4\ N\cdot m$$

$$\vec{u} \cdot \vec{v} = \underline{u_x v_x + u_y v_y + u_z v_z}$$

$$\vec{u} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$



$\vec{M}_A,\ M_A,\ M_y$

$$\vec{F} = 80\left[\frac{-30\hat{i} + 24\hat{j} + 12\hat{k}}{\sqrt{1620}}\right]$$

$$\vec{r} = 30\hat{i} - 6\hat{j}$$

$$\vec{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 30 & -6 & 0 \\ -30 & 24 & 12 \end{vmatrix} \frac{80}{\sqrt{1620}} = \frac{80}{18\sqrt{5}}\left[-72\hat{i} - 360\hat{j} + 540\hat{k}\right]$$

$$= -64\sqrt{5}\,\hat{i} - 320\sqrt{5}\,\hat{j} + 480\sqrt{5}\,\hat{k}$$

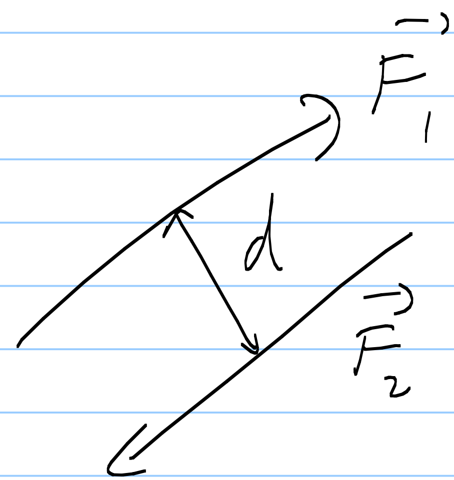
$M_A = 1298$ in-lbs

$M_y = -715.5$ in-lbs

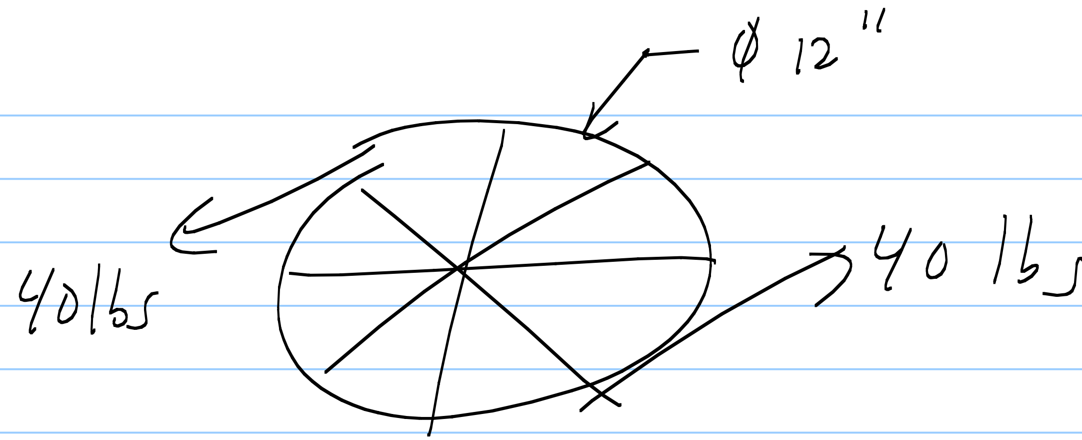
Couples

A couple is two parallel forces equal in magnitude and opposite in direction.

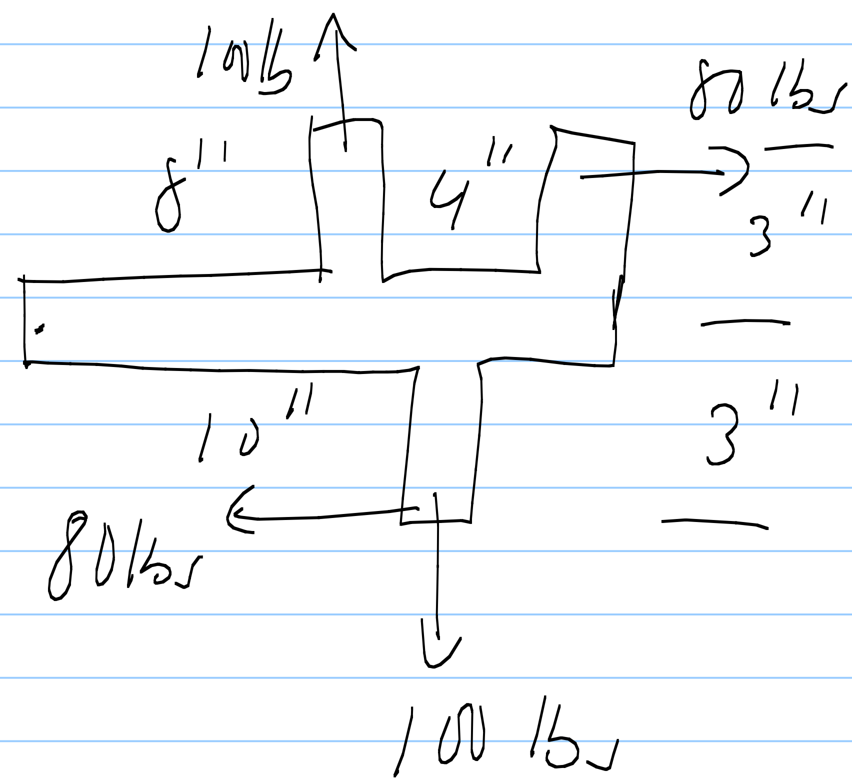
A pure moment



$$M = F_1 d$$



$M = 12 \cdot 40 = 480$ in-lbs



$M = -6 \cdot 80 - 2 \cdot 100$

$= -480 - 200$

$= -680$ in-lb