

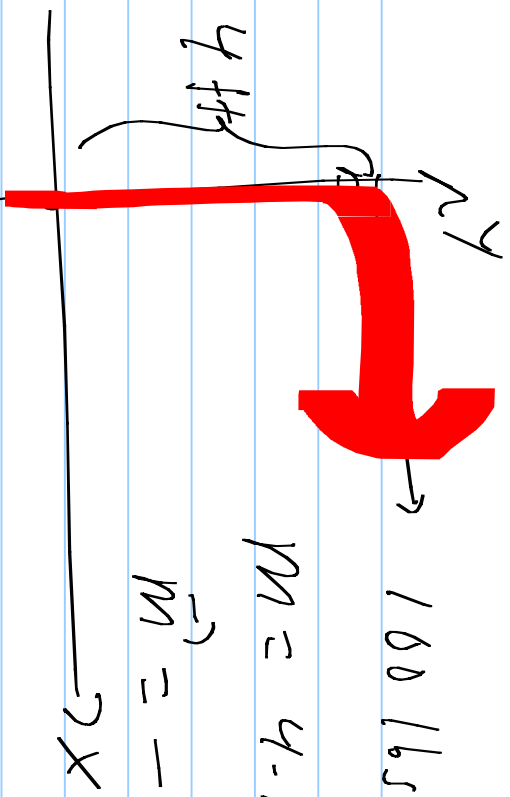
EGR 186

6/8/10

Rigid Bodies, Moments & Equivalent
Force Systems



$$M = F \cdot d$$



$$M = 4 \cdot 100 = 400 \text{ ft}\cdot\text{lbs}$$

$$\vec{M} = -400 \hat{k}$$

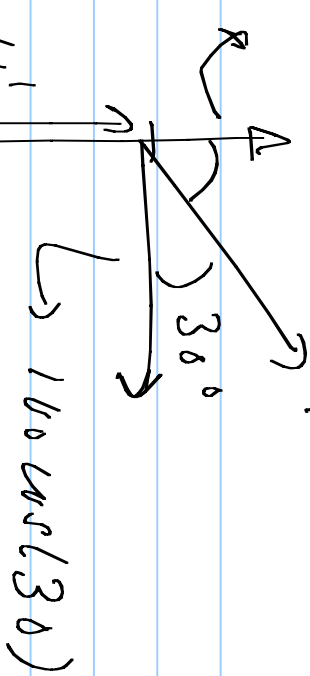
Moment is a vector quantity

Direction is always perpendicular to the plane containing \vec{r} & \vec{F}

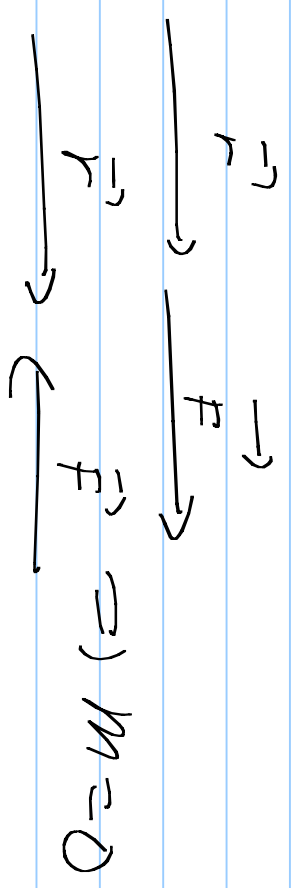
Use the RHR

$$100 \sin(30^\circ)$$

100 lb



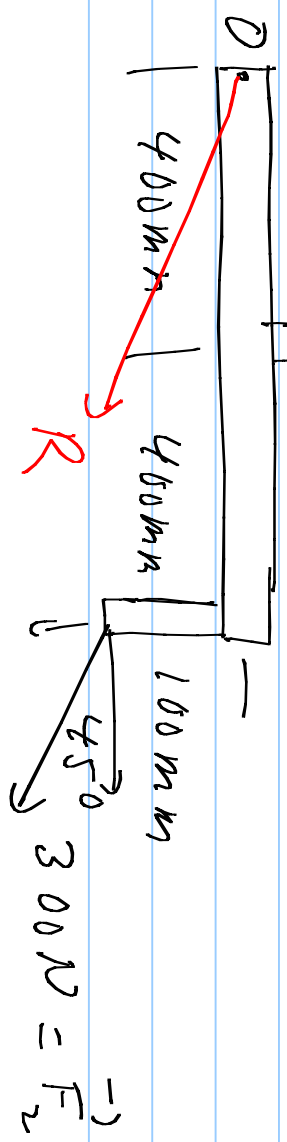
$$M = F \cdot d \sin \theta$$



$$M = 400 \cos(30^\circ)$$

$$= 200 \sqrt{3} = 346$$

$$2000 = F_1$$



$$3000 = F_2$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = (100\sqrt{3}\hat{i} + 100\hat{j}) + (150\sqrt{2}\hat{i} - 150\sqrt{2}\hat{j}) \\ = (100\sqrt{3} + 150\sqrt{2})\hat{i} + (100 - 150\sqrt{2})\hat{j}$$

$$R = 401 \text{ N}$$

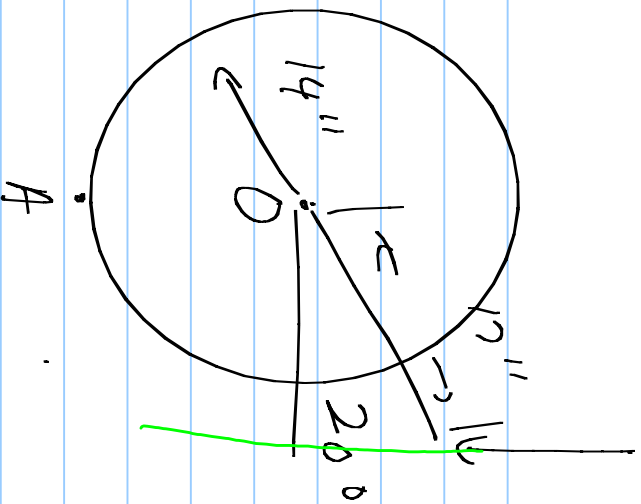
$$\theta = -16.2^\circ$$

$$\vec{F}_1 = 100\sqrt{3}\hat{i} + 100\hat{j}$$

$$M_1 = -100\sqrt{3}(1.1) + (100)(1.4) = 40 - 100\sqrt{3}$$

$$M_2 = 150\sqrt{2}(1.1) - (1.8)(150\sqrt{2}) = -105\sqrt{2}$$

$$M = M_1 + M_2 = -126 \text{ N}\cdot\text{m}$$



50 lbs

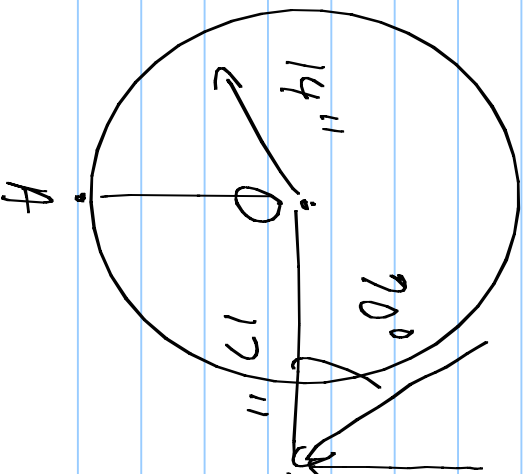
M_D, M_A

$$M_D = -50 \cdot 17 \cos(20) = -799 \text{ in-lbs}$$

$$= -66.6 \text{ ft-lbs}$$

$$M_A = -66.6 \text{ ft-lbs}$$

50 lbs



$$= 50 \sin 20$$

$$M_D = -17 \cdot 50 \sin(20)$$

$$= -799 \text{ in-lbs}$$

$$\hookrightarrow 50 \cos(70)$$

$$M_A = -799 - 14 \cdot 50 \cos 70$$

$$= -1038 \text{ in-lbs}$$

Vektor Repräsentation.

$$\Rightarrow \vec{M} = r \vec{x} F \vec{e}$$

$$|\vec{M}| = |\vec{r}| |F| \sin \theta$$

$$\vec{r} = x \vec{e}_i + y \vec{e}_j + z \vec{e}_k$$

$$\vec{F} = F_x \vec{e}_i + F_y \vec{e}_j + F_z \vec{e}_k$$

$$\vec{r}_1 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \hat{i} (yF_z - zF_y) - \hat{j} (xF_z - zF_x) + \hat{k} (xF_y - yF_x)$$

$$\vec{r}_1 = 2\hat{i} + 3\hat{j} - 4\hat{k} \quad (\text{m})$$

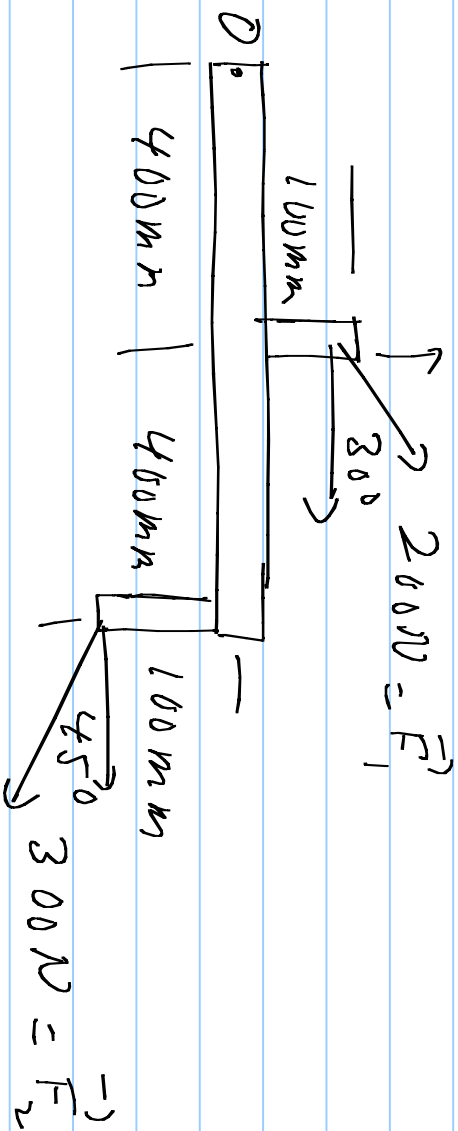
$$\vec{F} = 100\hat{i} + 100\hat{j} + 300\hat{k} \quad (\text{N})$$

$$\vec{r}_1 (1300)$$

$$\vec{r}_1 \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ 100 & 100 & 300 \end{vmatrix} = -\hat{j} (1000) + \hat{k} (-100)$$

$$\vec{M} = \vec{r} \times \vec{F} = 1300 \hat{i} - 1000 \hat{j} - 1000 \hat{k} \quad \text{N}\cdot\text{m}$$

$$|\vec{M}| = \left[(1300)^2 + (1000)^2 + (1000)^2 \right]^{1/2} = 1643 \text{ N}\cdot\text{m}$$



$$\vec{M} = (4 \hat{i} + 1 \hat{j}) \times (100\sqrt{3} \hat{i} + 100 \hat{j})$$

$$+ (1.8 \hat{i} - 1.1 \hat{j}) \times (150\sqrt{2} \hat{i} - 150\sqrt{2} \hat{j})$$

$$\vec{M}_1 = \begin{pmatrix} \vec{v} & \vec{J} & \vec{K} \\ .4 & .1 & 0 \\ 100\sqrt{3} & 100 & 0 \end{pmatrix} = \vec{K} (40 - 10\sqrt{3})$$

$$\vec{M}_2 = \begin{pmatrix} \vec{v} & \vec{J} & \vec{K} \\ .8 & -.1 & 0 \\ 150\sqrt{2} & -150\sqrt{2} & 0 \end{pmatrix} = \vec{K} (-120\sqrt{2} + 15\sqrt{2})$$

$$= -105\sqrt{2} \vec{K}$$

$$\vec{M} = [40 - 10\sqrt{3} - 105\sqrt{2}] \vec{K} = -126 \vec{K} \text{ N}\cdot\text{m}$$